Low-Emittance Tuning at CesrTA

Jim Shanks
Ph.D. Dissertation Defense
2013.07.17
• Motivation
• Emittance Tuning at CesrTA
• Simulations
• Emittance Tuning at CesrTA: Reprise
• Conclusions
• Motivation
• Emittance Tuning at CesrTA
• Simulations
• Emittance Tuning at CesrTA: Reprise
• Conclusions
• Emittance is the area of the beam in phase space
• Equilibrium between radiation damping and excitation:
  \[
  \left. \frac{d\varepsilon}{dt} \right|_{\text{damping}} = \sum_i \left. \frac{d\varepsilon}{dt} \right|_{\text{excitation}}
  \]
• Horizontal emittance determined by optics
• Lower bound for vertical emittance is dictated only by finite opening angle of radiation
  – In storage rings, \( \varepsilon_{y,\text{min}} \sim 0.2 \) pm
  – Vertical is significantly more susceptible to dilution
• This talk will cover vertical emittance in storage rings
• Light sources: Brightness

\[ B \propto \frac{1}{4\pi^2 y \sum H \sum V \sum H' \sum V'} \text{ Flux} \]

• Circular colliders: Luminosity

\[ \mathcal{L} \propto \left( 1 + \frac{\sigma_y}{\sigma_x} \right) \frac{1}{\sigma_y (\sigma_x + \sigma_y)} \]

\[ \sigma_{x,y} \propto \sqrt{\epsilon_{x,y}} \]

• International Linear Collider?
  – No transverse kicks \(\rightarrow\) No radiation \(\rightarrow\) No damping
  – Need damping rings!
• CESR-c converted to CesrTA
  – Dedicated testbed for low-emittance studies

• Why CESR?
  – Flexibility in optics
    • Electrons or positrons → Species dependence
    • Energy reach: 1.5-5.3GeV → Energy dependence
    • Lack of symmetry → Vary optics locally
  – Damping wigglers
  – Instrumentation
- Damping wigglers in zero-\( \eta \) straights
- \( \varepsilon_x = 2.6 \) nm (2.085 GeV)
- \( \varepsilon_y < 10 \) pm (target)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circumference [m]</td>
<td>768.4</td>
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<tr>
<td>Damping Wigglers</td>
<td>12</td>
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<tr>
<td>H / V Steerings</td>
<td>55/58</td>
</tr>
<tr>
<td>Quadrupoles</td>
<td>105</td>
</tr>
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<td>Skew Quadrupoles</td>
<td>27</td>
</tr>
<tr>
<td>RF Cavities</td>
<td>4</td>
</tr>
<tr>
<td>Position Monitors</td>
<td>100</td>
</tr>
</tbody>
</table>
• “Single-particle” effects
  – Magnet misalignments and field errors

• “Collective” effects
  – Electron cloud effect
  – Intra-beam scattering
  – Fast-ion instability
Survey and alignment:

Without further corrections: \( \rightarrow \) Need beam-based optics corrections!

<table>
<thead>
<tr>
<th>Element</th>
<th>Class</th>
<th>Error</th>
<th>RMS</th>
<th>Units</th>
</tr>
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<tbody>
<tr>
<td>Dipole</td>
<td>Roll</td>
<td>144</td>
<td>[\mu rad]</td>
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<tr>
<td>Quadrupole</td>
<td>y Offset</td>
<td>107.8</td>
<td>[\mu m]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Tilt</td>
<td>148</td>
<td>[\mu rad]</td>
<td></td>
</tr>
<tr>
<td>Wiggler</td>
<td>y Offset</td>
<td>250</td>
<td>[\mu m]</td>
<td></td>
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<tr>
<td></td>
<td>Tilt</td>
<td>300</td>
<td>[\mu rad]</td>
<td></td>
</tr>
</tbody>
</table>

\[ \epsilon_y \text{ [pm]} \]
• Motivation
• **Emittance Tuning at CesrTA**
• Simulations
• Emittance Tuning at CesrTA: Reprise
• Conclusions
• Computing corrections based on different combinations of basic beam-based measurements:
  – Orbit
  – Dispersion
  – Beta functions (betatron phase)
  – Coupling

• Beam Position Monitors (BPMs)
  – Bunch-by-bunch, turn-by-turn acquisition
  – Button-by-button readout
  – 300,000 bunch-turns buffer
  – 10 μm shot-to-shot reproducibility
• Many methods for combining these basic beam-based measurements into a correction

• Requirements for CesrTA optics correction:
  – Fast turnaround
  – Scales well to large rings
  – Routinely achieve $\varepsilon_y < 10$ pm
• Option 1: Response Matrix Analysis (RMA, ORM, LOCO)
  – Advantages:
    • Very complete characterization
    • Widely utilized
  – Drawbacks:
    • Slow
    • Does not scale well to large rings
• Option 2: Betatron phase / coupling  ➔ Use phase/coupling
  – Advantages:
    • Fast
    • Inherently scales well to large rings
  – Drawbacks:
    • Not as complete
    • Not widely used
1) Measure orbit
   – Correct using all 55 horizontal + 58 vertical steerings

2) Measure phase / coupling, horizontal dispersion
   – Correct using all 105 quadrupoles and 27 skew quadrupoles

3) Re-measure orbit, coupling, and vertical dispersion
   – Correct using all 27 skew quads and 58 vertical steerings

Turnaround time: ~15 minutes
• **Requirements:**
  – Bunch-by-bunch, turn-by-turn capability
  – High resolution: resolve $\varepsilon_y < 10$ pm
  – Dynamic range: 0.1-10 mA

• **Solution: x-ray beam size monitor**

\[
\sigma_y \approx \sqrt{\beta_y \varepsilon \sigma_{y,a}^2 + \sigma_{y,b}^2 + \sigma_{y,\eta y}^2}
\]

\[
\sigma_{y,a} = \sqrt{\epsilon_y \beta_y} \left[ \bar{C}_{22}^2 + \bar{C}_{12}^2 \right]^{1/2}
\]

\[
\sigma_{y,b} = \gamma \sqrt{\epsilon_b \beta_y}
\]

\[
\sigma_{y,\eta y} = \eta_y \frac{\sigma_E}{E}
\]
• X-ray Beam Size Monitor (xBSM)
  – Bunch-by-bunch, turn-by-turn imaging
  – \(2^{15}\)+ bunch-turns buffer
  – 1D “pinhole” imaging optic
### Results at CesrTA

<table>
<thead>
<tr>
<th>Energy [GeV]</th>
<th>Species</th>
<th>$\epsilon_y$ [pm]</th>
<th>$\delta\epsilon^\text{sys}_y$ [pm]</th>
<th>$\delta\epsilon^\text{stat}_y$ [pm]</th>
<th>Date</th>
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<tbody>
<tr>
<td>2.085</td>
<td>$e^+$</td>
<td>8.7</td>
<td>$+$2.9</td>
<td>$+$0.2</td>
<td>12/2012</td>
</tr>
<tr>
<td>2.085</td>
<td>$e^+$</td>
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<td>$+$3.3</td>
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<td>4/2013</td>
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<tr>
<td>2.085</td>
<td>$e^-$</td>
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<td>$+$3.4</td>
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<td>4/2013</td>
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<tr>
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<td>12.7</td>
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<tr>
<td>2.553</td>
<td>$e^+$</td>
<td>10.2</td>
<td>$+$2.9</td>
<td>$+$0.2</td>
<td>4/2013</td>
</tr>
</tbody>
</table>

- We achieved our emittance target
  - Could not reproduce in April 2013
  - Does not reproduce with electrons

- Could we do better? What’s holding us back?
  - Develop simulation suite to determine limitations of emittance correction
• Motivation
• Emittance Tuning at CesrTA
• Simulations
• Emittance Tuning at CesrTA: Reprise
• Conclusions
• ring_ma originally developed by Rich Helms (2008)

• Workflow:
  1. Introduce errors into a model lattice (offsets, tilts, …)
  2. Correct the optics through iterative correction:
     • Simulate “measurements” of beam optics
     • Compute corrections based on “measurements”
     • Record information about optics, emittance after each correction
  3. Repeat entire procedure for many lattices to build statistics

• Limitations:
  – Only able to misalign a few classes of magnets
  – Misalignment types are limited
  – Inaccurate modeling of BPM errors
  – No modeling of systematic effects of measurements
• Starting with ring_ma, add or expand:
  – Full ability to apply errors to (nearly) anything
    • Geometric: x/y/z offset, pitch/yaw/roll
    • Random multipole errors
    • Magnet calibration errors
  – Accurate modeling of BPM errors:
    • Shot-to-shot reproducibility
    • Geometry errors: offset, tilt, shear
    • Button-to-button relative gains
    • Button-by-button timing
  – Full simulation of measurement procedures:
    • Record button data and apply BPM measurement errors every turn
## Errors for ring_ma

<table>
<thead>
<tr>
<th>Element Class</th>
<th>Error</th>
<th>Applied RMS</th>
<th>Units</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dipole</td>
<td>$x$ Offset</td>
<td>0.9</td>
<td>[mm]</td>
<td>Survey</td>
</tr>
<tr>
<td></td>
<td>$y$ Offset</td>
<td>2.0</td>
<td>[mm]</td>
<td>Survey</td>
</tr>
<tr>
<td></td>
<td>$s$ Offset</td>
<td>2.3</td>
<td>[mm]</td>
<td>Survey</td>
</tr>
<tr>
<td></td>
<td>Roll</td>
<td>144</td>
<td>[µrad]</td>
<td>Survey</td>
</tr>
<tr>
<td></td>
<td>$x$ Pitch</td>
<td>600</td>
<td>[µrad]</td>
<td>Survey</td>
</tr>
<tr>
<td></td>
<td>$y$ Pitch</td>
<td>300</td>
<td>[µrad]</td>
<td>Survey</td>
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### Quadrupole

<table>
<thead>
<tr>
<th>Error</th>
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<th>Units</th>
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<tbody>
<tr>
<td>$x$ Offset</td>
<td>350</td>
<td>[µm]</td>
<td>Estimate</td>
</tr>
<tr>
<td>$y$ Offset</td>
<td>107.8</td>
<td>[µm]</td>
<td>Survey</td>
</tr>
<tr>
<td>$s$ Offset</td>
<td>5.2</td>
<td>[mm]</td>
<td>Survey</td>
</tr>
<tr>
<td>Tilt</td>
<td>148</td>
<td>[µrad]</td>
<td>Survey</td>
</tr>
<tr>
<td>$x$ Pitch</td>
<td>1100</td>
<td>[µrad]</td>
<td>Survey</td>
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<tr>
<td>$y$ Pitch</td>
<td>62</td>
<td>[µrad]</td>
<td>Survey</td>
</tr>
<tr>
<td>$k1$</td>
<td>0.1%</td>
<td>[%]</td>
<td>Estimate</td>
</tr>
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</table>

### Sextupole

<table>
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<th>Units</th>
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<tbody>
<tr>
<td>$x$ Offset</td>
<td>300</td>
<td>[µm]</td>
<td>Estimate</td>
</tr>
<tr>
<td>$y$ Offset</td>
<td>300</td>
<td>[µm]</td>
<td>Estimate</td>
</tr>
<tr>
<td>$s$ Offset</td>
<td>5.2</td>
<td>[mm]</td>
<td>Estimate</td>
</tr>
<tr>
<td>Tilt</td>
<td>200</td>
<td>[µrad]</td>
<td>Survey</td>
</tr>
<tr>
<td>$x$ Pitch</td>
<td>1200</td>
<td>[µrad]</td>
<td>Estimate</td>
</tr>
<tr>
<td>$y$ Pitch</td>
<td>800</td>
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<td>Estimate</td>
</tr>
<tr>
<td>$k2$</td>
<td>0.1%</td>
<td>[%]</td>
<td>Estimate</td>
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### Wiggler

<table>
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<th>Applied RMS</th>
<th>Units</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$ Offset</td>
<td>1</td>
<td>[mm]</td>
<td>Survey</td>
</tr>
<tr>
<td>$y$ Offset</td>
<td>250</td>
<td>[µm]</td>
<td>Survey</td>
</tr>
<tr>
<td>$s$ Offset</td>
<td>500</td>
<td>[µm]</td>
<td>Estimate</td>
</tr>
<tr>
<td>Tilt</td>
<td>300</td>
<td>[µrad]</td>
<td>Survey</td>
</tr>
<tr>
<td>$x$ Pitch</td>
<td>200</td>
<td>[µrad]</td>
<td>Estimate</td>
</tr>
<tr>
<td>$y$ Pitch</td>
<td>250</td>
<td>[µrad]</td>
<td>Estimate</td>
</tr>
</tbody>
</table>

## Multipole Values

<table>
<thead>
<tr>
<th>Element</th>
<th>Multipole</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Sextupole with</td>
<td>a3</td>
<td>$-7.25 \times 10^{-4}$</td>
</tr>
<tr>
<td>Vert. Steering</td>
<td>a5</td>
<td>$-1.46 \times 10^{-2}$</td>
</tr>
<tr>
<td></td>
<td>a7</td>
<td>$6.68 \times 10^{-4}$</td>
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<tr>
<td></td>
<td>a9</td>
<td>$8.7 \times 10^{-6}$</td>
</tr>
<tr>
<td></td>
<td>a11</td>
<td>$1.0 \times 10^{-5}$</td>
</tr>
<tr>
<td>Sextupole with</td>
<td>a4</td>
<td>$-1.2145 \times 10^{-1}$</td>
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<tr>
<td>Skew Quad Trim</td>
<td>a6</td>
<td>$2.16 \times 10^{-4}$</td>
</tr>
<tr>
<td></td>
<td>a8</td>
<td>$4.96 \times 10^{-4}$</td>
</tr>
<tr>
<td></td>
<td>a10</td>
<td>$-2.29 \times 10^{-5}$</td>
</tr>
<tr>
<td></td>
<td>a12</td>
<td>$-1.0 \times 10^{-5}$</td>
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### BPM Error

<table>
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<th>BPM Error</th>
<th>RMS</th>
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<tbody>
<tr>
<td>Reproducibility</td>
<td>10</td>
<td>[µm]</td>
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<tr>
<td>Tilt</td>
<td>12</td>
<td>[mrad]</td>
</tr>
<tr>
<td>Gains</td>
<td>0.5%</td>
<td>[%]</td>
</tr>
<tr>
<td>Timing</td>
<td>10</td>
<td>[ps]</td>
</tr>
<tr>
<td>Offset ($x, y$)</td>
<td>170</td>
<td>[µm]</td>
</tr>
<tr>
<td>$x$ Shear</td>
<td>100</td>
<td>[µm]</td>
</tr>
</tbody>
</table>
• Measured: $\varepsilon_y = 8.7$ pm
• Simulation: 95% of seeds achieved $\varepsilon_y < 4.1$ pm
  – More than $1\sigma$ difference
• Vertical dispersion consistent with measurements
• Difference in coupling accounts for $\Delta\varepsilon_y < 1$ pm

→ Discrepancy?
• Motivation
• Emittance Tuning at CesrTA
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• Conclusions
Recall: emittance is an equilibrium

\[
\frac{d\epsilon_y}{dt} \bigg|_{damping} = \sum_i \frac{d\epsilon_y}{dt} \bigg|_{excitation} = \sum_i \frac{\pi dE_{0i}}{4\gamma E^2_{i}H^2_{i}} \sum_{excitation} \frac{d\epsilon_y}{dt} \bigg|_{excitation}
\]

Contributions to emittance sum linearly

Many sources of excitation
  – Consider three main classes
Sources of Emittance Dilution

• **Optics**
  – Magnet misalignments
  – Emittance measurement errors
  – Inaccuracies in modeling

• **Collective effects**
  – Electron cloud
  – Intra-beam scattering
  – Fast-ion instability
  – Other

• **Non-static sources**
  – Magnet power supplies
  – RF system
  – Other
Sources of Emittance Dilution

• Optics
  – Magnet misalignments
  – Emittance measurement errors
  – Inaccuracies in modeling

• Collective effects
  – Electron cloud
  – Intra-beam scattering
  – Fast-ion instability
  – Other

• Non-static sources
  – Magnet power supplies
  – RF system
  – Other
• Known inaccuracies in modeling:
  – Long-range correlations / distortions
  – Manufacturing tolerances
    • Multipoles
    • Magnetic center vs. physical center
  – Misalignments in RF system
    • Offsets, pitch/yaw/tilt
  – Other?
Sources of Emittance Dilution

• **Optics**
  – Magnet misalignments
  – Emittance measurement errors
  – Inaccuracies in modeling → Possible

• **Collective effects**
  – Electron cloud
  – Intra-beam scattering
  – Fast-ion instability
  – Other

• **Non-static sources**
  – Magnet power supplies
  – RF system
  – Other
Sources of Emittance Dilution

• Optics
  – Magnet misalignments
  – Emittance measurement errors
  – Inaccuracies in modeling → Possible

• Collective effects
  – Electron cloud
  – Intra-beam scattering
  – Fast ion instability
  – Other → ?

• Non-static sources
  – Magnet power supplies
  – RF system
  – Other
• Collective effects: emittance will scale with current

→Possible, though unlikely
Sources of Emittance Dilution

• **Optics**
  – Magnet misalignments
  – Emittance measurement errors
  – Inaccuracies in modeling \(\Rightarrow\) Possible

• **Collective effects**
  – Electron cloud
  – Intra-beam scattering
  – Fast ion instability
  – Other \(\Rightarrow\) Possible

• **Non-static sources**
  – Magnet power supplies
  – RF system
  – Other
Sources of Emittance Dilution

• Optics
  – Magnet misalignments
  – Emittance measurement errors
  – Inaccuracies in modeling ➔ Possible

• Collective effects
  – Electron cloud
  – Intra-beam scattering
  – Fast ion instability
  – Other ➔ Possible

• Non-static sources ➔ ?
  – Magnet power supplies
  – RF system
  – Other
### RF affects emittance:

- Decreasing total RF voltage slightly lowers emittance
- “West RF only” → emittance increased by ~10%
- “East RF only” → emittance decreased by ~10%
Contributions to emittance will scale differently with energy

Examine four scenarios:

1. Assume no time-varying sources
2. Allow magnet power supply jitter
3. Allow RF voltage jitter
4. Allow both magnet power supply and RF voltage jitter
Sources of Emittance Dilution

• Optics
  – Magnet misalignments
  – Emittance measurement errors
  – Inaccuracies in modeling → Possible

• Collective effects
  – Electron cloud
  – Intra-beam scattering
  – Fast ion instability
  – Other → Possible

• Non-static sources → Possible
  – Magnet power supplies
  – RF system
  – Other
• Motivation
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Requirements for CesrTA optics correction:
- Fast turnaround ✓
- Scales well to large rings ✓
- Routinely achieve $\varepsilon_y < 10 \text{ pm}$ $\varepsilon_y < 12 \text{ pm}$ ($\varepsilon_{y,\text{min}} = 8.7 \text{ pm}$)

Simulations and analysis suggest $\varepsilon_y$ is not limited by:
- Static magnet errors: offsets, tilts, field errors, …
- Manufacturing tolerances: multipoles, magnetic centering
- Optics correction procedure
- BPM measurement errors
- xBSM measurement errors
- Collective effects: IBS, electron cloud, fast-ion
- RF voltage jitter
• Remaining candidates for vertical emittance dilution:
  – Misalignments in RF system
    • Offsets, pitch/yaw/tilt
  – Collective effects?
    • Wakefield?
    • RGS?
  – Non-static sources
    • RF system
    • RF input couplers
    • Magnet power supplies
    • Other?
• Thanks to everyone who made these studies possible
• Thanks for your attention!

Questions?
Backup Slides
### CesrTA Parameters

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<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>Circumference</td>
<td>768.4</td>
<td>[m]</td>
</tr>
<tr>
<td>Circulation Time</td>
<td>2.56</td>
<td>[µs]</td>
</tr>
<tr>
<td>Energy</td>
<td>2.085 (1.5-5.3)</td>
<td>[GeV]</td>
</tr>
<tr>
<td>Lattice Type</td>
<td>FODO</td>
<td></td>
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<tr>
<td>Symmetry</td>
<td>≈ Mirror</td>
<td></td>
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<tr>
<td>H / V Steerings</td>
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<td></td>
</tr>
<tr>
<td>Quadrupoles</td>
<td>105</td>
<td></td>
</tr>
<tr>
<td>Skew Quadrupoles</td>
<td>27</td>
<td></td>
</tr>
<tr>
<td>RF Cavities</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Max. Total RF Voltage</td>
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<td>[MV]</td>
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<tr>
<td>Wiggler $B_{max}$</td>
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<td>[T]</td>
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<tr>
<td>Position Monitors</td>
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<tbody>
<tr>
<td>Energy</td>
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<td>[GeV]</td>
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<tr>
<td>$\epsilon_x$</td>
<td>2.6</td>
<td>[nm]</td>
</tr>
<tr>
<td>$\epsilon_y$</td>
<td>&lt; 20</td>
<td>[pm]</td>
</tr>
<tr>
<td>$Q_{x,y}^{int}$</td>
<td>(14, 9)</td>
<td></td>
</tr>
<tr>
<td>$\langle \beta_{x,y} \rangle$</td>
<td>(16.3, 20.4)</td>
<td>[m]</td>
</tr>
<tr>
<td>$\langle \eta_x \rangle$</td>
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<td>[m]</td>
</tr>
<tr>
<td>Damping Time</td>
<td>50</td>
<td>[ms]</td>
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<tr>
<td>Energy Loss/Turn</td>
<td>189.2</td>
<td>[keV]</td>
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<td>Bunch Length</td>
<td>11</td>
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<tr>
<td>$\sigma_{E}/E$</td>
<td>$8.1 \times 10^{-4}$</td>
<td>[-]</td>
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<tr>
<td>$\alpha_p$</td>
<td>$6.8 \times 10^{-3}$</td>
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<table>
<thead>
<tr>
<th>Parameter</th>
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<tr>
<td># Poles</td>
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<tr>
<td>Wiggler Period</td>
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<tr>
<td>Pole Width</td>
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<tr>
<td>Pole Gap</td>
<td>7</td>
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<tr>
<td>Beam Pipe Aperture (H)</td>
<td>9</td>
<td>[cm]</td>
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<tr>
<td>Beam Pipe Aperture (V)</td>
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<td>[cm]</td>
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<tr>
<td>$B_{\text{max}}$</td>
<td>2.1</td>
<td>[T]</td>
</tr>
<tr>
<td>Field Rolloff at $x = \pm 20$ mm</td>
<td>1%</td>
<td>[%]</td>
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• Error propagation for vertical emittance

\[
\epsilon_y = \frac{\sigma_y^2 - \left( \eta_y \frac{\sigma_E}{E} \right)^2}{\beta_y}
\]

\[
\sigma_y^2 = \left( \frac{\sigma_{im}}{M} \right)^2 - \sigma_p^2
\]

\[
\delta \epsilon_y^{stat} = \sqrt{\left| \frac{\partial \epsilon_y}{\partial \beta_y} \right|^2 \left( \delta \beta_y^{stat} \right)^2 + \left| \frac{\partial \epsilon_y}{\partial \eta_y} \right|^2 \left( \delta \eta_y^{stat} \right)^2 + \left| \frac{\partial \epsilon_y}{\partial \sigma_{im}} \right|^2 \left( \delta \sigma_{im}^{stat} \right)^2}
\]

\[
\delta \epsilon_y^{sys} = \left| \frac{d \epsilon_y}{d \sigma_{im}} \right| \delta \sigma_{im} + \left| \frac{d \epsilon_y}{d \sigma_p} \right| \delta \sigma_p + \left| \frac{d \epsilon_y}{d s} \right| \delta s
\]

\[
\left| \frac{d \epsilon_y}{d s} \right| = \left| \frac{\partial \epsilon_y}{\partial \beta_y} \frac{\partial \beta_y}{d s} + \frac{\partial \epsilon_y}{\partial \eta_y} \frac{\partial \eta_y}{d s} + \frac{\partial \epsilon_y}{\partial M} \frac{\partial M}{d s} \right|
\]
• Reducing most-significant contributions:

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<td>170</td>
<td>148</td>
<td>144</td>
<td>250</td>
<td>1.86</td>
<td>4.15</td>
</tr>
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<td>144</td>
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• Reducing BPM errors:

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<td>10</td>
<td>100</td>
<td>1.86</td>
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Reducing all misalignments, errors by common factor

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<th>Reduction Factor</th>
<th>Mean $\epsilon_y$ [pm]</th>
<th>95%CL $\epsilon_y$ [pm]</th>
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<tr>
<td>Ideal Lattice</td>
<td>0.22</td>
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<td>1x (Nominal)</td>
<td>1.86</td>
<td>4.15</td>
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<td>2x</td>
<td>0.64</td>
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<td>4x</td>
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<td>10x</td>
<td>0.24</td>
<td>0.26</td>
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Turn-By-Turn Beam Size

FB Amplifier On

Run 49166d e+ 0.76 mA, Bunch 1

FB Amplifier Off

Run 49171d e+ 0.78 mA, Bunch 1

Run 49166d e+ 0.76 mA, Bunch 1

Run 49171d e+ 0.78 mA, Bunch 1
Turn-By-Turn Centroid Motion

FB Amplifier On

Run 49166d e+ 0.76 mA, Bunch 1

FB Amplifier Off

Run 49171d e+ 0.78 mA, Bunch 1