Measuring the Electric Dipole Moment of the Electron at Wilson Lab (Revised)

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Abstract

Of the various schemes described in an earlier version of this report for measuring the electric dipole moment (EDM) of the electron at Wilson Lab, only one survives as a realistic candidate. (One of the discarded schemes had been investigated earlier by Orlov et al.[12] for protons and shown not to work.) The proposed ring uses a 14.5 MeV electron beam in an all-electric ring with resonant longitudinal polarimetry. With spin tune $Q_s = 0$, the spin is frozen globally. Only single beam operation is considered, with, for example, one bunch. The bending field acting on the vertical EDM produces an accumulating, deviation-from-null, longitudinal polarization component. It is shown that there is neither risk nor cost uncertainty concerning the construction and commissioning at Wilson laboratory of such an all-electric storage ring. It would be patterned after the successful Brookhaven AGS Analogue Electron Ring. However, the precision with which the electron EDM can be measured remains uncertain. Rather than using simultaneously counter-circulating beams to cancel radial magnetic field errors (as has been proposed previously) here it is proposed to measure the counter-circulating beams sequentially rather than in colliding beam mode. This requires the development of highly accurate and highly stable digital nulling records at every BPM and accurate active emulation of the other beam. From statistical noise considerations alone, with (unrealistically) no allowance whatsoever for systematic errors, a five sigma deviation from zero determination of an electron EDM of 10^{-29} e-cm could be obtained in a year-long run. Systematic errors are expected to be as much as one hundred times greater, at least in a first generation of the experiment.

Contents

1 Introduction

Various schemes for using frozen spin storage rings to measure electric dipole moments (EDM) of various baryons have been investigated recently[1]. The most promising possibility continues to be measuring the EDM of the proton using an all-electric lattice in order to make simultaneously counter-circulating proton beams possible. An inexpensive first step in this direction would be to build, as a prototype, an all-electric, frozen-spin storage ring for electrons of kinetic energy $K = 14.5\,\mathrm{MeV}$ (which is the "magic energy" at which the electron spin is "frozen" because the "spin tune" Q_s^E vanishes). As well as serving as a prototype for an eventual proton ring, this electric ring could also be used to measure the electron EDM. Such a scheme has been described previously[2][3].

An earlier version of this report suggested the possiblity of measuring EDM's with spins frozen locally but not globally; i.e. spin tune Q_s equal to an integer other than zero. But, since no design has emerged making this possible, this option has had to be dropped.

1.1 Simplifying Assumptions

To simplify discussion I make some idealized assumptions, all of which will need to be revisited for estimating systematic errors. To reduce the need for repetitive qualifications concerning decoherence, I limit discussion to time intervals not greater than the spin coherence time (SCT). Roughly speaking, after taking account of the greater number of runs possible with small SCT values, the achievable precision will be proportional to the square root of SCT. For best precision one prefers a small number of long runs. For small SCT, one must make many short runs, and accept reduced precision. For now I also ignore accelerator physics complications that affect SCT, such as emittance growth, for example due to intrabeam scattering (IBS).

I also assume that we have complete control of the phase of precession around the vertical axis, at least on the average. To the extent this control is not achieved, some of the results in these notes will be invalid. Stability of less than 2π precession angle over 10 seconds has been exhibited for deuterons in the Juelich COSY ring[4] and active phase-locking in the near future appears promising[5]. For numerical estimates in this paper I assume SCT=1000 s.

"Frozen spin" operation amounts to "balancing on" the unstable equilibrium condition at an integer spin resonance. Such configurations are routinely investigated using "Froissart-Stora" scans in which the spin tune is varied at a controlled rate, slow or fast, across the resonance. There is secular radial precession and, eventually, for slow rate, a complete flip of the polarization. For the proposed EDM measurement the rate has to be made arbitrarily slow. If the electron EDM were huge it would dominate Froissart-Stora polarization reversals, making the EDM immediately measurable. But the electron EDM will be all but negligible compared to other effects capable of inducing Froissart-Stora polarization reversals. However, because the dominant rest frame electric and magnetic field are othogonal, the torques they cause acting on EDM and MDM are also orthogonal.

For any realistically-small EDM the EDM-induced precession angle will not exceed ten milliradians over runs of length comparable with the SCT. Even in the absence of other torques, this could not cause even a single polarization reversal. The best that can be hoped for is a measurably large change in the orientation of the beam polarization caused by the EDM. To be able to extract such an EDM effect requires all other resonance drivers to cause only exquisitely small "wrong symmetry" tipping of bunch polarizations—certainly small compared to π, which would correspond to a complete spin flip. But, with polarization reversals and multiple run repetitions, the "background" precession need not necessarily be smaller than the EDM-induced "foreground" precession

I also ignore (without justification at present) possibly limiting phenomena, such as spurious response of the resonant polarimeter to the electrical fields of the beam bunches rather than to their magnetic moments.

2 All-Electric Lattices for Electron EDM Measurement

The basic all-electron EDM ring suggested for Wilson Lab is shown in Figure 1. Possible later generation rings are shown in an appendix.

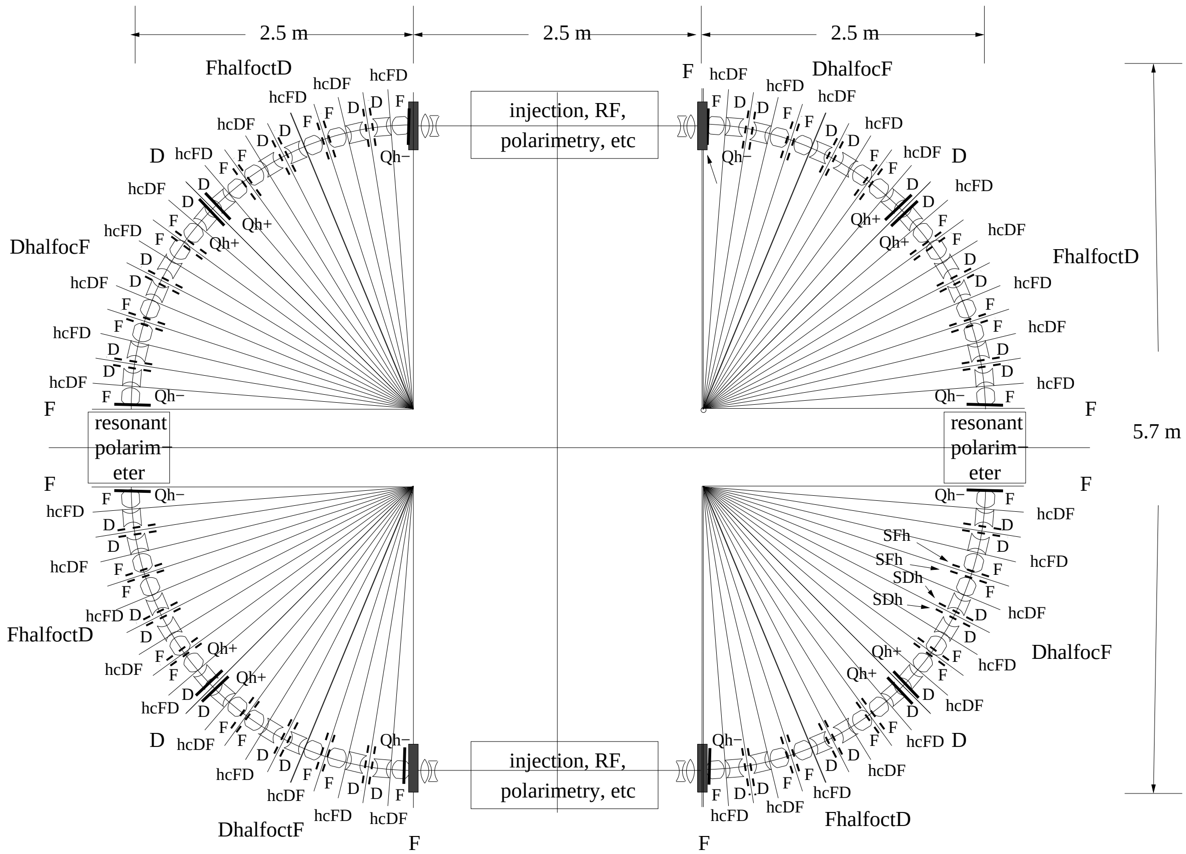


Figure 1: Basic all-electric, electron EDM storage ring.

3 Polarimetry

3.1 Polarimetry Strategies

Various polarimetry strategies have been considered for the proton EDM measurement. A scheme using proton-carbon elastic scattering has been considered promising[4]. With counter-circulating beams, p-p scattering is another possibility[2]. Both of these measure transverse polarization. In an earlier note[2] I have described a resonator that responds to longitudinal bunch polarization and can therefore be used to measure longitudinal polarization. The fact that the electron MDM is greater than the proton MDM by a factor roughly equal the ratio of their masses makes resonant polarimetry far more promising for electrons than for protons. (On the other hand, e-e colliding beam scattering polarimetry is less promising for electrons than is p-p scattering for protons.) These statements are explained more fully in separate reports.

Torques due to MDM's are huge compared to any achievable electric dipole moment induced torque. Any scheme to measure an EDM will have to exploit "deviation from null" signal detection. That is, the configuration has to be arranged such that (ideally) the MDM causes zero signal in a channel in which the EDM gives a measurably-large signal. For frozen-spin protons in an all-electric lattice, with spin frozen forward, there is no intentional induced vertical polarization component except that caused by the EDM. In the absence of systematic error any measurably-large vertical polarization accumulation is then ascribed to the proton EDM. Proton-carbon or p-p polarimetry, because they are sensitive to transverse polarization, can be used for this measurement. Such scattering polarimeters are “fast”, meaning they measure the polarization of every circulating bunch. But their precision is subject to unfavorable counting statistics, especially because any realistic asymmetry is proportional to the difference of nearly equal counting rates.

3.2 Resonant Polarimetry

Though continuing to keep scattering polarimetry in mind, this paper instead uses resonant longitudinal polarimetry. Relying on resonance, a resonant polarimeter is “slow”—incapable of resolving individual bunches. The resonator can only measure the net polarization of whatever bunches there are, circulating CW and/or CCW. As already stated, the eventual EDM signal has to be a deviation from null. To be useful the resonator requires the summed longitudinal polarizations to vanish initially (and, in the absence of EDM, indefinitely). This constrains the possible configurations of bunch polarizations.

The fact that a resonant polarimeter sums over all circulating bunches has two substantial advantages over scattering polarimeters. The obvious advantage is that the resonant polarimeter is non-destructive; it does not attenuate the beams. But a more significant advantage is that the resonant polarimeter sums phasor amplitudes while scattering polarimeters sum (or rather subtract) intensities (in the form of counting rates)—the precision of left-right or up-down scattering asymmetries are necessarily limited by counting statistic errors on two approximately equal rates that have to be subtracted.

The resonant longitudinal polarimeter has no such problem. (Ignoring possible spurious response due to bunch electric fields) there is zero response to unpolarized beams. This applies to the total, or net, or phasor sum of longitudinal polarizations, even if there are multiple polarized bunches in counter-circulating beams, provided the phasor sum vanishes. In principle, to the extent it is caused by magnetic dipole moments, this phasor sum can be made to vanish. Any observed accumulating signal would then be ascribed to electron EDM. In practice there will inevitably be error effects which give signals in the resonator polarimeter comparable to or larger than the EDM signal. Improving the precision of the EDM measurement will amount to beating down these sources of systematic error. The EDM polarimetry precision will also be limited by thermal (and other) noise. But these errors are subject to reduction by averaging over multiple runs. In my note *Magnetic Resonance and Beam-Beam Polarimetry for Frozen Spin Storage Rings,* dated October 13, 2012 (available upon request, or from the EDM website) I described the use of a high-Q resonator, shown in Figure 2, to measure the proton EDM. This application is challenging for protons, especially because it requires very low temperature cryogenics. Even then, the signal to noise ratio for a one hundred percent longitudinally polarized beam of nominal intensity was only about 50 to 1. This would correspond to a longitudinal polarization noise floor of about 20 mr tilt of a 100 percent polarized beam.

This method will be far more effective for electrons since the electron magnetic moment is three orders of magnitude higher than the proton MDM. I have not yet updated the resonator notes, neither to give the parameters for the electron beam resonator, nor to show the Microwave Studio plots and calculations produced by Valery Shemelin for the resonator geometry shown here.

My previous estimates of polarimeter signal to noise (for protons) have assumed $N_e = 10^{10}$ particles per bunch. Because of uncertainty concerning electron operation and beam-beam interaction (though the present proposal does not, in fact, have colliding beams) and IBS, with the quite soft 14.5 MeV electrons, I have reduced this to $N_e = 10^9$ electrons per bunch. One anticipates, then, a noise floor of perhaps 1 milliradian in the sensitivity to longitudinal tilt of the electron beam polarization.

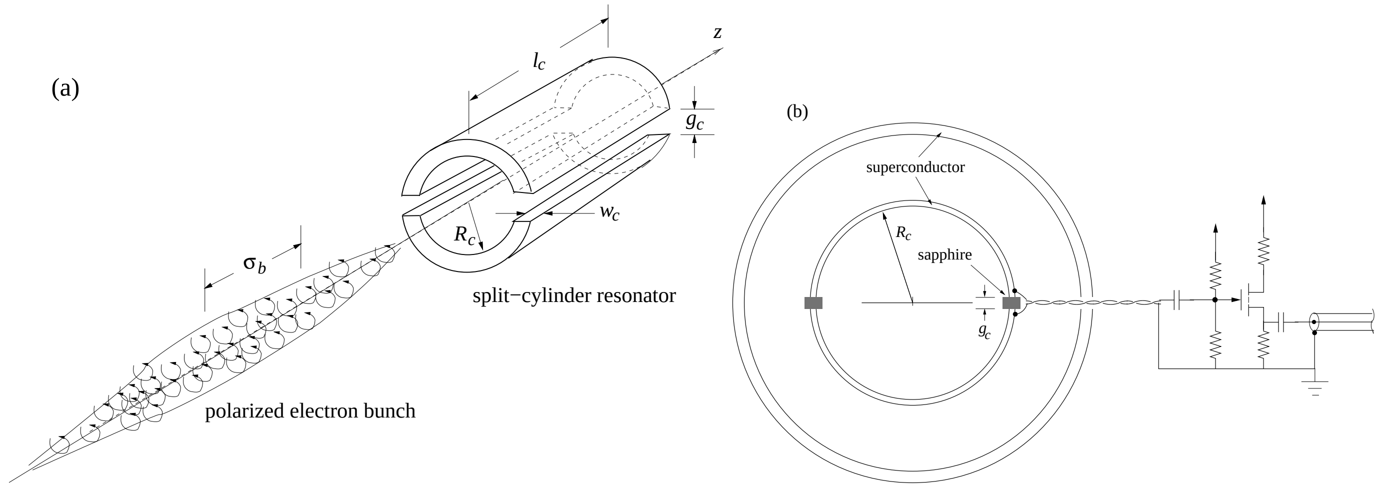


Figure 2: (a) Perspective view of the polarimeter. Dimensions are defined for the polarized electron bunch and the superconducting split-cylinder resonator. The resonator gap, shown as vacuum in the figure, will actually be filled by a low loss saphire spacer. (b) End view of polarimeter and readout (copied from reference [10]) using a low temperature pHEMT transistor such as Agilent type ATF-35143 in a source follower circuit. The gap height is greatly exaggerated; it will only be tens of micrometers even with a high permittivity spacer such as sapphire.

4 Spin Precession in the Arcs and RF Cavities

4.1 Field Transformations

The dominant fields in a horizontal storage ring are radial lab frame electric $-E\hat{\mathbf{x}}$ and/or vertical lab magnetic field $B\hat{\mathbf{y}}$. They give[13] electron rest frame field vectors $\mathbf{E}'$ and $\mathbf{B}'$;

$$\mathbf{E}' = \gamma(\mathbf{E} + \boldsymbol{\beta} \times c\mathbf{B}) = -\gamma(E + \beta cB)\,\hat{\mathbf{x}} \tag{1}$$

$$\mathbf{B}' = \gamma(\mathbf{B} - \boldsymbol{\beta} \times \mathbf{E}/c) = \gamma(B + \beta E/c)\,\hat{\mathbf{y}}. \tag{2}$$

Longitudinal electric and magnetic components are related by

$$E'_z = E_z, \tag{3}$$

$$B'_z = B_z. \tag{4}$$

A striking feature of the longitudinal formulas is that a purely longitudinal electric field in the laboratory is purely longitudinal in the electron rest frame. Such a field acts on the electric, but not the magnetic, dipole moment of the electron. This seems promising for any EDM measurement which relies on RF-cavity-applied electric field to alter the electron momentum dynamics. However at least two RF cavities would be required, since the longitudinal boosts have to sum to zero, for fixed energy storage ring operation. This also means that, even in storage ring operation, the beam energy has to be different in different sectors of the ring. This, in turn, requires the bend fields to be different in different sectors. Y. Orlov, W. Morse, and Y. Semertzidis[12] have shown that this form of velocity modulation produces no measurable EDM signal. Another noteworthy feature of the formulas is that, unlike the transverse components, there is no γ-enhancement factor in the rest frame longitudinal fields, relative to the lab frame. For low energy protons, because γ is close to 1, this distinction is insignificant. But for electrons γ is very large and the torque applied by E_z can normally be neglected in practice.

4.2 Comparison of EDM and MDM-Induced Precession

There is a curious near-coincidence in the precession of the spins of electrons and protons as they follow circular paths in a magnetic field. Though the electron's magnetic moment is three orders of magnitude greater than the proton's, its anomalous magnetic moment is three orders of magnitude less. Their perceived precessions are therefore roughly equal. Numerically the values of magnetic moments and anomalous magnetic moments are:

$$\begin{aligned}
\text{electron magneton} \equiv \mu_e &= 5.78838175 \times 10^{-5}\,\text{eV/T},\\
G_e &= 0.001159652,\\
\frac{G_e\mu_e}{\hbar} &= 1.020 \times 10^{8}\text{s}^{-1}/\text{T}\\
\text{proton magneton} \equiv \mu_p &= 3.1524512 \times 10^{-8}\,\text{eV/T},\\
G_p &= 1.792847356,\\
\frac{G_p\mu_p}{\hbar} &= 0.859 \times 10^{8}\text{s}^{-1}/\text{T}
\end{aligned} \tag{5}$$

This near equality of precession rates in magnetic fields is convenient for comparing electron and proton EDM setups.

For both protons and electrons we define a **nominal EDM of** 10^{-29} ***e*****-cm** by the product $d_{\text{nom}}c$ (because E and B have different units).

$$\frac{d_{\text{nom}}c}{\hbar} = \frac{10^{-29} \times (0.01) \times 3 \times 10^{8}}{6.58 \times 10^{-16}} = 4.56 \times 10^{-8}\,\text{s}^{-1}/\text{T}. \tag{6}$$

We then determine the ratio of the precessions in a magnetic field caused by an EDM of this magnitude to the just-evaluated MDM precessions. For electrons the relative-effectiveness ratio is

$$\rho^{(e)}_{\text{EM}} = \frac{4.56 \times 10^{-8}}{1.020 \times 10^{8}} = 0.46 \times 10^{-15}. \tag{7}$$

The corresponding ratio for protons is not very different. Measuring EDM's can then be separated into two tasks:

Relative precession task: Distinguish EDM-induced precession from spurious, wrong-plane, MDM-induced, precession. (Especially for electrons) this is probably the dominant source of EDM measurement error.

Absolute precession task: For a pure Dirac particle circulating in a storage ring magnetic field the precession is 2π per turn. With revolution frequency of 10 Mhz, this is 6.283×10^{7} r/s. Applying the $\rho^{(e)}_{\text{EM}}$ ratio, for globally frozen spin one has to plan on measuring a "nominal" EDM-induced precession of order 3×10^{-8} r/s, or about 3 mr/day. For the anticipated 1 mr longitudinal polarization sensitivity this is about three sigma per day, for an EDM of 10^{-29} e-cm. These estimates will be updated below for actual lattice parameters.

The current upper limit inferred for the electron EDM (corrected up from the atomic EDM by three orders of magnitude to account for induced polarization) is about 100 in our nominal 10^{-29} e-cm EDM units. These estimates suggest that the absolute electron EDM precession is amply large to be measureable to considerably better accuracy than the present limit. How spurious MDM precession can be held to this same precision will require much study and research.

4.3 MDM-Induced Precession in an Electric Guide Field

A particle in its rest system, with angular momentum $\mathbf{s}'$ and magnetic dipole moment $g\mu_B\mathbf{s}'$, in magnetic field $\mathbf{B}'$, is subject to torque $g\mu_B\mathbf{s}' \times \mathbf{B}'$. By Newton's angular equation, substituting from Eq. (2) to express the fields in laboratory coordinates,

$$\frac{d\mathbf{s}'}{dt'} = -g\mu_B\mathbf{B}' \times \mathbf{s}' = -g\mu_B\gamma(B + (\beta E/c))\,\hat{\mathbf{y}} \times \mathbf{s}'. \tag{8}$$

Customarily the rest frame angular momentum is represented by $\mathbf{s}$ rather than by $\mathbf{s}'$ and the laboratory time interval $dt = dt'\gamma$ is used instead of rest frame time interval dt'. Also, the magnitude $|\mathbf{s}|$ is known to be constant.

With the laboratory field being purely electric and purely radial, the vertical component of $\mathbf{s}$ is conserved. The normalized horizontal component $\hat{\mathbf{s}}$ satisfies

$$\frac{d\hat{\mathbf{s}}}{dt} = -\frac{g}{2}\, 2\mu_B(\beta E/c))\hat{\mathbf{y}} \times \hat{\mathbf{s}}. \tag{9}$$

As Jackson explains[14], relativistic effects cause the electron axis to precess in the laboratory, irrespective of any static moments the electron may have. This Thomas precession causes the polarization vector to precess even if there is no torque acting on the magnetic or electric moments. This precession has to be allowed for when ascribing precession to MDM's or EDM's. In our case, the beam direction advances uniformly, by 2π during one revolution period T_{lab} and the Thomas precession term is

$$\begin{aligned} \frac{\gamma-1}{v^2}\Big(\frac{d\mathbf{v}}{dt}\times\mathbf{v}\Big)\times\hat{\mathbf{s}} &= \frac{\gamma-1}{v^2}\Big(\frac{-\hat{\mathbf{x}}v^2}{r_0}\times\mathbf{v}\Big)\times\hat{\mathbf{s}} \\ &= \frac{\gamma-1}{r_0/v}\,\hat{\mathbf{y}}\times\hat{\mathbf{s}} = \Big(1-\frac{1}{\gamma}\Big)\,2\mu_B\,\frac{E/c}{\beta}\,\hat{\mathbf{y}}\times\hat{\mathbf{s}}. \end{aligned} \tag{10}$$

Adding this term to the electric part on the rhs of Eq. (8),

$$\frac{d\hat{\mathbf{s}}}{dt}\Big|_{\text{MDM,E}} = 2\mu_B\Big(-\frac{g}{2}+\frac{\gamma}{\gamma+1}\Big)\,\beta\frac{E}{c}\,\hat{\mathbf{y}}\times\hat{\mathbf{s}}. \tag{11}$$

This agrees with Jackson's Eq. (11.170). We try a solution of the form

$$\hat{\mathbf{s}}_{\text{MDM,E}} = \big(\hat{\mathbf{Z}}\,\cos Q_{\text{MDM,E}}\frac{v}{r_0}t - \hat{\mathbf{X}}\,\sin Q_{\text{MDM,E}}\frac{v}{r_0}t\big). \tag{12}$$

For circular motion in an electric field, $r_0E = \beta(pc/e)$, which leads to

$$Q_{\text{MDM,E}} = G\beta^2\gamma - \frac{1}{\gamma} + 1. \tag{13}$$

An accelerator spin physicist would define the "spin tune" Q_s in an electric field by

$$Q_E \equiv \frac{d\alpha}{d\theta}\Big|_E = G\beta^2\gamma - \frac{1}{\gamma}, \tag{14}$$

where α is the angle between spin vector and particle velocity. These two definitions of "tune" have therefore been inconsistent. Unlike the beam direction, which advances by 2π each turn, and would therefore be described as a "tune" value of 1, the spin tune is reckoned relative to the particle velocity rather than relative to a frame fixed in the laboratory. This accounts for the "+1" on the rhs of Eq. (13).) The content of this section has therefore amounted to being a derivation of Eq. (14), for the spin tune in an electric ring.

4.4 EDM-Induced Precession in an Electric Guide Field

A particle at rest, with angular momentum $\mathbf{s}$ and electric dipole moment $d\,\mathbf{s}$, in electric field $\mathbf{E}'$, is subject to torque $d\,\mathbf{s}\times\mathbf{E}'$. Using Eq. (1), if the laboratory field is purely electric, the induced precession satisfies

$$\frac{d\mathbf{s}}{dt}\Big|_{\text{EDM,E}} = d\,\mathbf{s}\times\mathbf{E}' = -d\,\mathbf{s}\times E\,\hat{\mathbf{x}}. \tag{15}$$

This precession is small enough to be treated as a perturbative addition to otherwise-inexorable polarization evolution.

To calculate EDM-induced precession we can use evolution formulas to describe dependence on θ, the beam angle, which advances by 2π every turn. With $(\mathbf{X},\mathbf{Y},\mathbf{Z})$ being Cartesian coordinates fixed in the lab, setting $Q_{\rm MDM,E}=1$, the Frenet and spin vectors advance as

$$\hat{\mathbf{x}} = \hat{\mathbf{X}}\,\cos\theta + \hat{\mathbf{Z}}\,\sin\theta,$$
$$\hat{\mathbf{s}} = \hat{\mathbf{Y}}\cos\Theta + \big(\hat{\mathbf{Z}}\cos\theta - \hat{\mathbf{X}}\sin\theta\big)\sin\Theta. \tag{16}$$

Here Θ is the polar angle of the polarization relative to the vertical axis. Substituting these on the right hand side of Eq. (15) the evolution of $\mathbf{s}_{\rm EDM,E}$ is the driven response given by

$$\frac{v}{r_0}\,\frac{d\hat{\mathbf{s}}}{d\theta}\Big|_{\rm EDM,E} = -d\;E\,\Big(\cos\Theta(-\hat{\mathbf{Z}}\cos\theta + \hat{\mathbf{X}}\sin\theta) + \sin\Theta\big(\cos\theta\big)\hat{\mathbf{Y}}\Big) \tag{17}$$

As discussed earlier, it is convenient to express the EDM precession as a ratio to the MDM precession. From here on the EDM will be expressed as $\tilde{d}$ which is in units of 10^{-29} e-cm For spin tune $Q_s=0$ the MDM precession rate is equal to the orbit revolution rate $\omega_{\rm rev}$. The EDM precession rates is then

$$\omega_{\rm EDM,E} = \tilde{d}\,\rho^{(e)}_{\rm EM}\omega_{\rm rev}, \tag{18}$$

with $\rho^{(e)}_{\rm EM}$ given in Eq. (7).

4.5 The Leading Spurious, EDM-Mimicking Precession Sources

The main bending field, which is electric, has magnitude E. Other fields will be denoted by ΔE or ΔB.

For an apparatus intended to measure EDM's, one has to be prepared to suppress any MDM-induced precession that mimics EDM-induced precession. The leading source of systematic error is (unintended) rest frame radial magnetic field B'_x which, acting on the MDM, mimics the effect of radial electric field acting on the EDM. It is only the rest frame magnetic field $\mathbf{B}'$ that causes MDM-induced precession but it is necessary to keep track of both laboratory frame components ΔB_x and ΔE_y. These are also the lab frame field components capable of causing the closed orbit to deviate from the ideal, horizontal, design plane. Because the guide field is radial electric, the dominant field error can be expected to be a vertical laboratory electric component ΔE_y. Transformed to the electron rest frame, this produces a radial magnetic field component $\Delta B'_x$.

To the extent the beam *does* move out of the horizontal plane the cross product in the transformation from lab to rest frame can produce a radial magnetic field. This I neglect, relying on precise storage ring construction, and hoping that the tendency to average to zero will not be defeated by conspiring correlations. Substituting into Eqs. (2), the fields being retained are

$$\mathbf{E}' = \gamma(\Delta E_y\hat{\mathbf{y}} \pm \beta\hat{\mathbf{z}}\times c\Delta B_x\hat{\mathbf{x}}) = \gamma(\Delta E_y \pm \beta c\Delta B_x)\hat{\mathbf{y}}, \tag{19}$$
$$\mathbf{B}' = \gamma(\Delta B_x\hat{\mathbf{x}} \mp \beta\hat{\mathbf{z}}\times\Delta E_y\mathbf{y}/c) = \gamma(\Delta B_x \pm \beta\Delta E_y/c)\hat{\mathbf{x}} \tag{20}$$

Here the upper of the $\pm$ and $\mp$ signs refers to a clockwise (CW) beam and the lower to a counter-clockwise (CCW) beam. In the rest frame, since the velocity vanishes, only the first of these equations influences the vertical motion of the particle by the Lorentz force,

$$\frac{dp'_y}{dt'} = -e\gamma(\Delta E_y \pm \beta c\Delta B_x). \tag{21}$$

Here ΔE_y and ΔB_x are unknown lab frame field errors, and we are temporarily ignoring the fact that ΔB_x causes CW and CCW orbits to differ. Knowing that the beam stays more or less centered vertically over long times, averaging this equation yields the result

$$\langle\Delta E_y\rangle = \mp\beta c\langle\Delta B_x\rangle. \tag{22}$$

In particular, if $\langle \Delta B_x \rangle = 0$ then $\langle \Delta E_y \rangle = 0$.

(This analysis has neglected gravitational forces. There is certainly a vertical laboratory gravitational force $(m_e\gamma)g$ acting on each electron corresponding to its total energy $\gamma m_e c^2$. Neglecting the issue of difference between CW and CCW beams, one can define an "equivalent" laboratory magnetic field $\Delta B_x^{\rm G-equiv}$ such that

$$ev\Delta B_x^{\rm G-equiv} = m_e\gamma g, \quad \text{or} \quad \Delta B_x^{\rm G-equiv} = \frac{\gamma m_e g}{ec} = 5.5 \times 10^{-18}\,\text{T}. \tag{23}$$

Since this is far smaller that the smallest conceivable uncertainty in the true magnetic field error ΔB_x, I will continue to neglect gravity[1].)

We will concentrate on a more-or-less vertically polarized beam, with rest frame polarization vector $\mathbf{s}$ (conventionally written without a prime in spite of relating to the rest frame) given by

$$\mathbf{s} = (s_y + \Delta s_y)\,\hat{\mathbf{y}} + \Delta s_x\,\hat{\mathbf{x}} + \Delta s_z\,\hat{\mathbf{z}}. \tag{24}$$

In this dominant vertical polarization configuration, the EDM effect to be detected is a longitudinal torque, which is proportional to the z-component of $\mathbf{B}' \times \mathbf{s}$,

$$\begin{aligned}\mathbf{B}' \times \mathbf{s}|_z &= \Big|\gamma(\Delta B_x \pm \beta \Delta E_y/c)\hat{\mathbf{x}} \times \Big((s_y + \Delta s_y)\,\hat{\mathbf{y}} + \Delta s_x\,\hat{\mathbf{x}} + \Delta s_z\,\hat{\mathbf{z}}\Big)\Big|_z \\ &\approx \gamma(\Delta B_x \pm \beta \Delta E_y/c)(s_y + \Delta s_y). \end{aligned} \tag{25}$$

Averaging this equation, and applying Eq. (22), we obtain

$$\langle \mathbf{B}' \times \mathbf{s}\rangle|_z = \gamma(1-\beta^2)\langle \Delta B_x \rangle (s_y + \Delta s_y) = \frac{1}{\gamma}\,\langle \Delta B_x \rangle (s_y + \Delta s_y). \tag{26}$$

This can be compared to the corresponding intentional, EDM-induced torque constant-of-proportionality, which is

$$\langle \mathbf{E}'/\mathbf{c} \times \mathbf{s}\rangle|_z = -\gamma(E/c)(s_y + \Delta s_y). \tag{27}$$

Comparing Eqs. (26) and (27), the disrimination against spurious precession can be expressed by the "discrimination factor",

$$\text{D.F.} = \frac{\langle \Delta B_x \rangle}{\gamma^2 E/c}. \tag{28}$$

4.6 Spurious Precession Caused by $\langle \Delta B_x \rangle$ Magnetic Field Error

After ΔE_y the leading laboratory error field will be ΔB_x. This field violates the time reversal symmetry and causes the CW and CCW beams to separate vertically. This separation will be limited however by the opposite electric field components $E_y^{\pm}$ that the beams encounter because of their different orbits. Transformed to the electron rest frame, these yield

$$\mathbf{E}'^{(\pm)} = \gamma((\Delta E_y + \Delta E_y^{\pm})\hat{\mathbf{y}} \pm \beta\hat{\mathbf{z}} \times c\Delta B_x \hat{\mathbf{x}}) = \gamma((\Delta E_y + \Delta E_y^{\pm}) \pm \beta c \Delta B_x)\hat{\mathbf{y}} \tag{29}$$

$$\mathbf{B}'^{(\pm)} = \gamma(\Delta B_x \hat{\mathbf{x}} \mp \beta \hat{\mathbf{z}} \times (\Delta E_y + \Delta E_y^{\pm})\mathbf{y}/c) = \gamma(\Delta B_x \pm \beta(\Delta E_y + \Delta E_y^{\pm})/c)\hat{\mathbf{x}} \tag{30}$$

Here ΔE_y and ΔB_x are the same lab frame field errors as before and $\Delta E_y^{\pm}$ are laboratory frame electric fields that develop (differently CW and CCW) to limit the orbit deviations caused by

[1] I am told that work by Orlov, Flanagan, and Semertzidis (which I have not attempted to understand) indicates that general relativity significantly alters the interpretation of EDM storage ring experiments. Unlike the purely classical $g = 9.8\,\text{m/s}^2$ gravitational acceleration accounted for here, to be of concern, such an effect must imply a general relativistic effect capable of mimicking an observable CP-violating effect. It seems likely to me that any such mechanism could plausibly be responsible for the observed matter/anti-matter imbalance that has motivated the search for non-vanishing EDM's in the first place. By this reasoning, general relativistic considerations cannot really alter the motivation for attempting to measure electric dipole moments. In other words, *if it looks like CP violation, it is CP-violation.*

ΔB_x. Averaged over longitudinal coordinate s the fields $\Delta E_y^{\pm}$ will be opposite for CW and CCW beams but the equality is not guaranteed locally at every longitudinal position. (This is because the field errors do not respect the lattice mirror symmetries. As it happens, since the EDM lattice will have only mild beta function dependence on s the symmetry will be only mildly broken, causing the fields $\Delta E_y^{\pm}$ to be more or less equal and opposite locally.)

The rest frame vertical electric fields are

$$E'^{(+)}_y = \gamma((\Delta E_y + \Delta E_y^+) + \beta c \Delta B_x), \tag{31}$$
$$E'^{(-)}_y = \gamma((\Delta E_y + \Delta E_y^-) - \beta c \Delta B_x). \tag{32}$$

Upon averaging these yield

$$\langle \Delta E_y + \Delta E_y^+ \rangle = -\langle \beta c \Delta B_x \rangle, \tag{33}$$
$$\langle \Delta E_y + \Delta E_y^- \rangle = \langle \beta c \Delta B_x \rangle\rangle. \tag{34}$$

The rest frame magnetic fields are

$$B'^{(+)}_x = \gamma(\Delta B_x + \beta(\Delta E_y + \Delta E_y^+)/c), \tag{35}$$
$$B'^{(-)}_x = \gamma(\Delta B_x - \beta(\Delta E_y + \Delta E_y^-)/c). \tag{36}$$

Upon averaging and substituting from Eqs. (33) and (34) these yield

$$\langle B'^{(+)}_x \rangle = \frac{\langle \Delta B_x \rangle}{\gamma}, \tag{37}$$
$$\langle B'^{(-)}_x \rangle = \frac{\langle \Delta B_x \rangle}{\gamma}. \tag{38}$$

These formulas are consistent with Eq. (26). They also show that the precession caused by $\langle \Delta B_x \rangle$ field error is the same for CW and CCW beams. One cannot, therefore, hope to cancel the spurious $\langle \Delta B_x \rangle$-induced signal by subtracting CW and CCW results.

If the radial magnetic field causes a net-upward force on an electron beam then its effect on a counter-circulating electron beam will be downward. One can therefore measure $\langle \Delta B_x \rangle$ by measuring the vertical separation between counter-circulating beams. There are two possibilities:

If the beams circulate simultaneously the beam position monitors can function in differential mode, enabling the separation to be measured with best-possible, noise-immune precision. One cannot easily be sure, however, that colliding beam operation does not bring in other sources of systematic error. This report will refer to running in colliding beam mode as a "second-generation" possibility.

It is possible that measuring the vertical separation consecutively is as good, or better than, measuring simultaneously-circulating beams. Because the same beam can be injected alternately in the two directions, this option is inexpensive. But it requires the comparison of absolute BPM measurements made in the two orientations. The limitation to EDM precision following this route is estimated next.

The vertical deflection at BPM d caused by N_a vertical deflections $\Delta y'_a$ at lattice positions a is given by[18]

$$\frac{y_d}{\sqrt{\beta_d}} = \sum_{a=1}^{N_a} \frac{\cos(\mu_y/2 - \phi_{da})}{2\sin(\mu_y/2)} \sqrt{\beta_a}\, \Delta y'_a, \tag{39}$$

where $\mu_y = 2\pi Q_y$ is the lattice tune, ϕ_{da} is the vertical betatron phase advance from a to d, and β_a and β_d are the corresponding vertical Twiss function values. For the EDM ring the β-functions will not depend strongly on position (especially if mini-beta are not required to reduce beam-beam interaction of counter-circulating beams). This, simplifies Eq. (39) to

$$y_d = \sum_{a=1}^{N_a} \frac{\cos(\mu_y/2 - \phi_{da})}{2\sin(\mu_y/2)} \beta_a\, \Delta y'_a. \tag{40}$$

To increase rejection of radial magnetic field error ΔB_x, the vertical tune Q_y will be adjusted intentionally so that $Q_y << 1$. Assuming the N_a deflection errors are distributed more or less uniformly around the ring, an average value for the cosine factor is $\sin(Q_y\pi)/(Q_y\pi)$. The vertical deflection error $\Delta y'_a$ caused by magnetic field error $\Delta B_{x,a}$ in an element of length L_a is

$$\Delta y'_a = \frac{c}{p_0 c/e}\,\Delta B_{x,a} L_a, \tag{41}$$

and the summation gives approximately

$$\sum_{a=1}^{N_a} \Delta y'_a \approx \frac{c}{p_0 c/e}\,\langle \Delta B_x \rangle\, 2\pi r_0, \tag{42}$$

Also $\beta_y \approx r_0/Q_y$. With these approximations, Eq. (40) becomes

$$y_d \approx \frac{r_0^2}{Q_y^2}\,\frac{c}{p_0 c/e}\,\langle \Delta B_x \rangle. \tag{43}$$

The momentum factor can be expressed in terms of E as $p_0 c/e = r_0 E$, yielding

$$y_d \approx \frac{r_0}{Q_y^2}\,\frac{\langle \Delta B_x \rangle}{E/c}. \tag{44}$$

The main purpose for the N_d vertical BPM's is to accurately measure the average deflection y_d (or rather, the difference in vertical deflections of counter-circulating beams) in order to infer $\langle \Delta B_x \rangle$. Assuming the precision of each such measurement is σ_y, the r.m.s. field error is given by

$$\frac{\sigma_{B_x}}{E/c} = \frac{1}{\sqrt{2N_d}}\,Q_y^2\,\frac{\sigma_y}{r_0}. \tag{45}$$

This ratio to E form is especially convenient because the effect of $\langle \Delta B_x \rangle$ is to apply a spurious torque mimicking the EDM torque provided by electric field E. Substituting from Eq. (45) into Eq. (28) and using parameter values to be spelled out shortly,

$$\text{D.F.} = \frac{1}{\gamma^2}\,\frac{1}{\sqrt{2N_d}}\,Q_y^2\,\frac{\sigma_y}{r_0} \overset{\text{e.g.}}{=} 0.44 \times 10^{-13}. \tag{46}$$

as the discrimination factor against spurious MDM-induced precession. This is the factor to be applied against the advantage factor $\rho^{(e)}_{EM} = 0.46 \times 10^{-15}$ the MDM has over an EDM value of 10^{-29} e-cm. This measure suggests the smallest upper bound that can be set on the electron EDM will be about 10^{-27} e-cm. (This happens to be approximately the presently-claimed upper bound.) But, in a later section, I will give reasons for regarding this estimate to be too pessimistic.

A priori one does not know σ_y, but $\sigma_y \approx 100\,$nm BPM resolution has been achieved, for example in connection with the Interanational Linear Collider[19]. To minimize D.F. one clearly needs to minimize Q_y. In recent EDM studies, such as [1], a nominal value of $Q_y = 0.1$ has been used for a proton EDM ring. For a 14.5 MeV electron ring of radius $r_0 = 2.5\,$m, the equilibrium horizontal emittance would be about $10^{-11}\,$m and the vertical smaller still. But these emittances are unrealistically small, since the damping time will be long compared to the storage time. Let us suppose therefore that a round beam with normalized emittances $\epsilon_n = 3\,$mm-mr and (therefore) geometric emittances $\epsilon = 10^{-7}\,$m is injected and captured with no emittance growth. With the Twiss beta function β_y more or less constant, its value is approximately r_0/Q_y, which becomes large as Q_y becomes small. The r.m.s. beam height is given by $\sqrt{\epsilon\beta_y} = \sqrt{10^{-7} \times (2.5/Q_y)}$. The vertical aperture, taken to be 3 cm, should be about ten sigma for good beam lifetime. By this estimate Q_y could be as small as 0.027. For now, to simplify comparison with previous (mainly proton) EDM proposals, I will continue to use the far more conservative value of $Q_y = 0.1$. Taking $\sqrt{2N_d}$=10, and $\gamma = 30$, we obtain the numerical D.F. value given with Eq. (46).

Previous studies have assumed that counter-circulating beams would be simultaneously present in the storage rings. This seems to me to be, at best, unnecessary and, at worst, simply counter-productive—beam-beam interactions have the potential for making systematic errors greater, which would be counter-productive. The rationale for having counter-circulating beams is to enable the BPM's to measure deviation-from-null, differential signals, and assumes the counter-circulating beam currents are equal. For this to be accurately null, the beam currents have to be equalized to high precision.

Better than fighting colliding beam problems, it seems to me, is to develop precision electronics at every beam position monitor which records (with high accuracy) the nulling current with one circulating beam. The subsequent run uses this record for nulling the BPM signal against an (emulated) counter-circulating beam having (to very high absolute accuracy) the same beam current. This careful signal treatment can be expected to maintain the level of precision assumed in Eq. (46).

4.7 Cancellation of Longitudinal Magnetic Field Errors

In the proposed initial Wilson lab ring there are no intentional longitudinal field components B_z. But there will be bend field errors ΔB_z. There will therefore have to be longitudinal trim fields $\Delta B_z^{\rm trim}$. Some candidates as second generation rings (for example as shown in an appendix) have longitudinal magnetic fields strong enough to produce Siberian snakes. Such fields, though huge, will here be subsumed into $\Delta B_z^{\rm trim}$. The rest frame longitudinal fields to be discussed are then

$$\mathbf{B}' = (\Delta B_z + \Delta B_z^{\rm trim})\,\hat{\mathbf{z}}, \tag{47}$$

$$\mathbf{E}' = E^{\rm rf}\,\hat{\mathbf{z}}, \tag{48}$$

where ΔB_z represents an *unknown* longitudinal field component, and $\Delta B_z^{\rm trim}$ is a *known* longitudinal trim field. It is unecessary to introduce a time-dependent magnetic RF term, because the RF electric field does not interact (on-axis) with the MDM. (Off-axis there is a time-dependent magnetic field that has be discussed separately, along with other issues such as RF misalignment.) Various non-zero contributions to $\mathbf{E}'$ have also been suppressed, since they give only EDM-induced precession small compared to the dominant EDM effect.

We are concentrating on more or less vertically polarized beams, with rest frame polarization vector $\mathbf{s}$ given by

$$\mathbf{s} = (s_y + \Delta s_y)\,\hat{\mathbf{y}} + \Delta s_x\,\hat{\mathbf{x}} + \Delta s_z\,\hat{\mathbf{z}}. \tag{49}$$

MDM and EDM torques are proportional, respectively, to the cross products

$$\begin{aligned}\mathbf{B}' \times \mathbf{s} &= (\Delta B_z + \Delta B_z^{\rm trim})\,\hat{\mathbf{z}} \times \big((s_y + \Delta s_y)\,\hat{\mathbf{x}} - \Delta s_x\,\hat{\mathbf{y}}\big) \\ &= (\Delta B_z + \Delta B_z^{\rm trim})\big((s_y + \Delta s_y)\,\hat{\mathbf{y}} + \Delta s_x\,\hat{\mathbf{x}}\big) \end{aligned}\tag{50}$$

$$\begin{aligned}\mathbf{E}' \times \mathbf{s} &= E^{\rm rf}\,\hat{\mathbf{z}} \times \big((s_y + \Delta s_y)\,\hat{\mathbf{x}} - \Delta s_x\,\hat{\mathbf{y}}\big) \\ &= E^{\rm rf}\,\big((s_y + \Delta s_y)\,\hat{\mathbf{y}} + \Delta s_x\,\hat{\mathbf{x}}\big). \end{aligned}\tag{51}$$

The only important torque here is magnetic since the electric RF torque is neglible compared to the bend field torque given in Eq. (27). The magnetic torque can, potentially, be large but the leading vertically-directed precession $(\Delta B_z + \Delta B_z^{\rm trim})\big((s_y + \Delta s_y)\,\hat{\mathbf{y}}$ will be cancelled by balancing on the unstable equilibrium point in a Froissart-Stora scan, producing

$$\Delta B_z^{\rm trim} = -\langle \Delta B_z \rangle. \tag{52}$$

Since both ΔB_z and ΔB_x contribute to unbalancing the equilibrium, though at right angles, they both have to be adjusted empirically. If the B_z trimming altered $\mathbf{E}' \times \mathbf{s}$ it would give an unknown EDM effect. But there is no such coupling.

4.8 "Vernier" Extraction of the True EDM Signal

The discussion accompanying Eq. (46) provided one estimate of the ultimate possible precision that can be achieved in the measure of the electron EDM. This is not necessarily the last word, however, since there may be ways of obtaining better precision. This possibility will be pursued by the following *gedanken experiment.*

Let us say, following the earlier estimate, that we have achieved an upper limit of $d_{\rm max} = 1.0\times 10^{-27}$ e-cm, based on a measured precession angle corresponding to longitudinal polarization component $\Delta s_z^{\rm meas.}$. By our assumptions this precession is due to some unknown sum of true EDM-induced precession and spurious ΔB_x-MDM-induced induced precession. $\Delta s_z^{\rm meas.}$ was the extent to which our best tuning efforts failed to keep the beam balanced in its condition of unstable equilibrium. The claimed upper limit $d_{\rm max} = 1.0\times 10^{-27}$ e-cm ascribes this entirely to the EDM.

Perhaps there is a way of ascribing some fraction of the residual precession to the MDM? A so-called "Wien Filter" can achieve this, provided it does not, itself, introduce new unknown errors. Let us pretend we have such a perfect, error-free device, and worry about *its* error fields later.

A Wien filter consists of superimposed electric and magnetic fields with opposite electric and magnetic deflections resulting in no net deflection. The absence of deflection in the laboratory implies the absence of electric field $\mathbf{E}'$ in the electron rest frame. This, in turn, implies the absence of interaction with the electric dipole moment of the electron. This gives the Wien filter the capability of altering EDM-mimicking precession without actually causing any EDM-induced precession. Assuming perfect reproducibility (as we have been doing) then, repeating the run, we can, this time, use the Wien filter, with dead-reckoned strength, to tune away the residual $\Delta s_z^{\rm meas.}$ precession. Here "dead-reckoned strength" assumes, for example, the entire signal seen previously was spurious. After this revision the residual precession would be interpreted as a true EDM effect. (Under our idealized assumptions) this residual signal has to be smaller than before, even in the unlikely event that the spurious signal was previously *subtracting* from the true signal.

Introducing a magnetic element into a previously all-electric system has the potential for completely fouling up the experiment, especially if its electric and magnetic fields are very strong. (The data collecting sequence described so far has at least minimized this complication by asking the Wien filter to provide only a miniscule precession.) The exact cancellation of electric and magnetic deflections (like balancing a bump in a storage ring) can only be confirmed by the exquisite precision of the BPM system. It is not clear whether this bump balancing requirement places a more or a less strict precision of the BPM system. Here I have assumed that the BPM's are perfect, meaning they have been used previously to tune the Wien filter to have no influence on the closed orbit.

A Wien filter has the further property of having reversed fields when switching between CW and CCW beam. Likely to be its most important error fields will be stray magnetic which, unlike static magnetic fields, reverse with the beam direction, making them subject to determination using CW and CCW beams.

One supposes that the master ΔB_x correction is supplied uniformly around the ring. With the Wien filter we then have a "dual" correction scheme, which is, potentially, an improvement over uniform correction. As mentioned earlier, though, this has only been a *gedanken experiment.* Its inherent errors have not be investigated, and the extent to which it woould actually improve the precision not calculated. All that has been demonstated, therefore, is that the earlier estimate of EDM measurement precision may be too pessimistic.

A more important implication is that it is *reproducibility* which will determine the ultimately achievable precision. If nothing changes (except orientation) in switching between CW and CCW then, with time, systematic errors can be beaten down arbitrarily. Even small time-dependent changes can be averaged over except to the extent they are (inevitably) correlated with beam orientation.

5 Ring Setup and Estimated Electron EDM Precision

There has been no discussion so far of the precision with which element parameters and positions need to be determined. This has been deferred so that the analysis of the storage ring as a "trap" could be emphasized. Much of the ultimate EDM precision is governed primarily by average parameters, but establishing all ring parameters to the best possible precision will surely be required for ultimate precision.

Spurious precession estimates so far have negected the "wrong components" resulting when the design orbit does not lie in a single, presumeably horizontal, plane. To justify this neglect will require this planar closed orbit requirement to be religiously respected.

For crude estimates let us take $10\,\mu$m as the default precision with which all positioning and lengths are to be established. In some cases this may be unnecessarily precise. In cases where it is insufficiently precise, the practicality of achieving higher precision will have to be established. Fortunately some of the most important parameters can be established to far greater precision by frequency measurements.

Using conventional surveying techniques the ring circumference $\mathcal{C}$ can be established with a fractional accuracy in the range from 10^{-6} to 10^{-5}. Being a frequency, the revolution frequency $\nu_{\rm rev}$ can be determined to much higher accuracy; let us regard $\nu_{\rm rev}$ as arbitrarily accurate. The electron central energy can also be obtained to almost arbitrarily high accuracy, by measuring the frequency of a $Q_s = Q_e\gamma$ integer depolarization resonance[20]. Since the imprecision of β is quadratic in the imprecision of γ, the particle speed is determined with absurdly high accuracy. This also determines the ring circumference $\mathcal{C}$ to correspondingly high accuracy.

5.1 Ultimate Statistical Precision

Many of the accelerator operations required for the success of the EDM measurement have already been demonstrated in routine accelerator operation, especially at BNL and at the COSY ring in Juelich Germany. In particular the vertical polarization in rings with vertical magnetic fields is routinely preserved almost indefinitely. But this does not guarantee the long spin coherence time that will be needed for the EDM measurement. This is because any actual EDM-induced polarization will be normal to the beam polarization and will itself be subject to a fairly short SCT.

The operation required for the EDM measurement that *has not been demonstrated operationally* is the ability to balance indefinitely on an integer spin resonance. This is precisely the situation that is studiously avoided in ordinary operations. The feasability of balancing on a point of unstable equilibrium is a fundamental uncertainty about the proposed method. It will surely require active feedback. For now we assume this balancing is possible.

The true EDM effect will be to produce polarization precession at some rate $\Omega_{\rm EDM}$ such that the tipping angle of the polarization vector in time dt is $\Omega_{\rm EDM}\,dt$. This tipping of a normalized vertical spin vector manifests itself as the polarization Δs_z surviving at time T, which can be written as an integral over polarization produced at earlier times t as attenuated by decoherence;

$$\Delta s_z(T) = \Omega_{\rm EDM}\int_0^T e^{-\frac{(T-t)^2}{2\tau_d^2}}\,dt = \Omega_{\rm EDM}\tau_d\,\frac{\pi}{2}\,{\rm erf}\Big(\frac{T}{\sqrt{2}\tau_d}\Big)$$
$$\approx \begin{cases} 0.83\,\Omega_{\rm EDM}T & 0 < T < 1.5\tau_d \\ 1.25\,\Omega_{\rm EDM}\tau_d & 1.5\tau_d < T. \end{cases} \tag{53}$$

where τ_d is a decoherence time. (For early times decoherence is Gaussian-like rather than the more naively-expected exponential decay[21].) For present purposes it is good enough to say that runs of length τ_d will produce polarization tipping of approximately $\Omega_{\rm EDM}\tau_d$, and that there is nothing to be gained by extending runs longer than τ_d.

As mentioned earlier, if the set-up can survive periodic polarization reversals then, by reversing the polarization after regular intervals of duration τ_d, decoherence can be postponed indefinately while providing an alternating polarization signal of amplitude $\Omega_{\rm EDM}\tau_d$. Should this be successful a very clean tentative EDM signal will become available in time, irrespective of the noise floor in the polarimeter. (This would also be true for a proton EDM measurement, for which noise in the polarimeter is far more of an issue.)

It will be the comparison of this signal with reversed polarization, reversed circulation direction, and (in later generations of the experiment) interchange of particles and anti-particles that will provide the final EDM measurement. Calculating the systematic errors in these comparisons is well beyond our present capabilities.

What can be estimated though, if the irreducible polarimeter noise level and the SCT are assumed known, is the EDM precision limit imposed by absolute rate and random error considerations. We have assumed $\tau_d = 1000\,$s as the spin coherence time and $\sigma_{\rm polarimeter} = 10^{-3}$ as the rms noise level of the polarimeter. The circumference of the proposed ring is 18.8 m and its angular revolution rate is $\omega_{\rm rev} = 2\pi \times 1.59 \times 10^7 = 10^8\,$r/s. From Eq. (18), the spin precession for one data collction interval, in units of the rms noise, is

$$\frac{\omega_{\rm EDM,E}\tau_d}{\sigma_{\rm polarimeter}} = \frac{\tilde{d} \times (10^8) \times (0.46 \times 10^{-15}) \times 10^3}{10^{-3}} = 0.046\,\tilde{d}. \tag{54}$$

The most optimistic assumption possible is that the polarimeter reads the accurate EDM precession with no subtractions and that no calibrations whatsoever are needed. Running continuously for 10^7 seconds, with runs of length 10^3 seconds, and alotting zero time for injection and set-up, one will perform 10^4 measurements. This will give a statistical error 100 times smaller than can be obtained in a single run of duration equal to the spin coherence time. 10^4 runs will produce a $4.6\,\sigma$ measurement for an electron EDM value of 10^{-29} e-cm. Of course these estimates are wildly overly-optimistic as regards the absence of systematic errors.

An EDM value of 10^{-29} e-cm still exceeds standard model estimates by two orders of magnitude. To obtain another factor of ten in statistical precision could be obtained, for example, by factors of ten improvement in both spin coherence time and polarimeter noise level, neither of which can be ruled out as plausible goals.

I have profited especially from conversations with Bill Morse, Yuri Orlov, Frank Rathmann, Yannis Semertzidis, and John Talman, as well as with Mei Bai, Ivan Bazarov, Mike Blaskowitz, Peter Cameron, Bruce Dunham, Ralf Gebel, Tom Kinoshita, Andreas Lehrach, Alfredo Luccio, Nikolay Malitsky, Bob Meller, Michael Peskin, Dave Rubin, Yuri Senichev, Valeri Shemelin, and Hans Stroeher.

A Resurrecting the AGS Analogue Electron Ring

A.1 Introduction

The electric dipole moment (EDM) of the proton is so small that, in order to reduce systematic errors enough to measure it in a storage ring requires measuring differences between counter-circulating beams. For the counter-circulating orbits to be identical requires the bending field to be electric. It is only fortuitous that the anomalous magnetic moment of the proton is such that the proton spins can be "frozen" (i.e. rotating such that they remain fixed relative to the momentum) for protons "trapped" in an all-electric storage ring. This occurs only at the "magic" moment of 0.7 GeV/c, which is sufficiently low for electrical bending to be practical.

All-electric accelerator rings are rare. The ring most nearly resembling the ring that will be required for the proton EDM experiment is the "Electron Analogue" ring proposed in Brookhaven in August of 1953 in connection with the AGS proton ring which was already under construction mainly to investigate particle loss in passing through transition. By July, 1955 the analog ring had been approved, built, and commissioned, and had achieved its intended purpose.

The present paper describes my (still only partially complete) attempt to "resurrect" this ring design from historic BNL documentation, and simulate its performance with codes intended for the proton EDM experiment, in particular John Talman's UAL/ETEAPOT code[8].

A quite superficial (less than day long) search of the BNL library and the Accelerator-Collider report library produced one quite extensive report, produced retrospectively in 1991 by Martin Plotkin[16], and a few ancient reports analysing machine studies results. The two main inputs for the present investigation are contained in Section A.2. The first, from [16], provides enough detail to reconstruct the lattice, the second, from a report by Ernest Courant[17], contains the experimental data to be simulated.

After comparing our simulation results to Courant's results, this paper proceeds to give other ETEAPOT simulation results and to interpret them theoretically. Transfer matrices produced by ETEAPOT have unity determinants and therefore satisfy Liouville's theorem exactly. And they are derived from orbits that are exactly Hamiltonian and symplectic (in the analytic dynamics sense). Nevertheless, because the coordinates are not canonical, the transfer maps are not exactly symplectic in the linear algebraic sense. This complicates, but does not prevent, a quite compact analytic eigenmode and normal mode partioning needed for synchro-betatron analysis. This is explained in appendices.

A.2 Historical BNL Documents

The "Conceptual Design Report" for the AGS Analogue electron ring was a three page letter from BNL Director Haworth to A.E.C. (predecessor of D.O.E) Director of Research Johnson, applying for funding. The letter, almost in its entirety, is reproduced on the following three pages. Then the main result contained in Ernest Courant's report[17] is copied in Figure 6. It plots points of observed beam loss and observed beam disruption in the (Q_x, Q_y) tune plane and correlates them with expected resonances. Beam loss occurs on integer resonances, beam disruption occurs on half integer resonances. This report on and analysis of data collected in machine studies less than two years after the submission of the proposal, would be judged *more than respectable* by modern standards. Figure 8, later in this paper, simulates these results using TEAPOT.

August 21, 1953

Dr. T.H. Johnson, Director
Division of Research
U.S. Atomic Energy Commission
Washington 25, D.C.

Dear Tom:

This letter concerns certain aspects of our accelerator development program, particularly the proposed electron model.

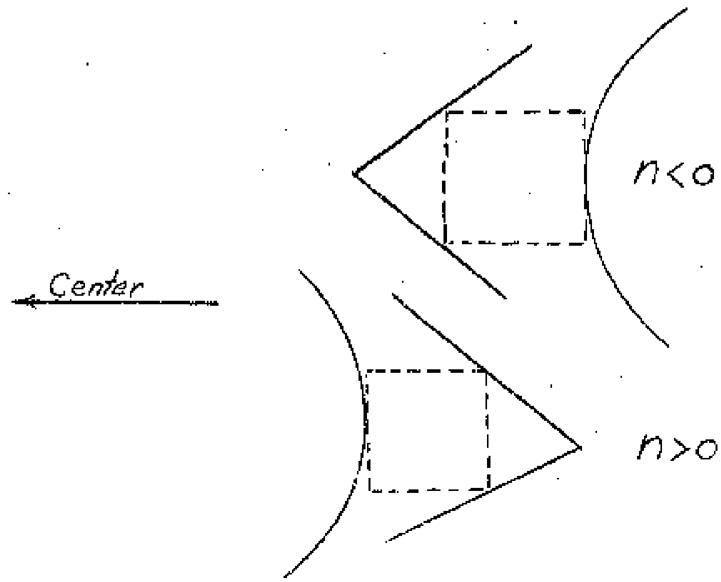
As you know, the general development of a very high energy alternating gradient synchrotron is proceeding actively at Brookhaven, utilizing operating funds allocated to Basic Physics Research. As I explained in my letter of August 12, however, these funds are insufficient to carry forward the development as rapidly as desirable. Also, there are certain steps which should be taken for which the expenditure of operating funds is not appropriate. The first and most important of these is the construction of an electron model intended to provide final assurance of the technical feasibility of the chosen machine and, more importantly, to provide information enabling us to design in the most effective and economical manner. (We have no doubt of the general feasibility of accelerators of this type.)

We have given considerable thought to the requirements for such a model and to the philosophy which should guide us in designing and building it. In the alternating gradient synchrotron, two problems require especially careful exploration by extensive calculation and experimental modelling. These are the close-spaced resonances in the betatron oscillations and the shift of phase stability at intermediate energies. It seems best to study these problems with an electron accelerator which would be essentially an analogue rather than an exact model. This device should, in our opinion, be designed to yield the maximum of orbital data with a minimum of engineering complications, especially those not applicable to a final machine. After considerable thought we have arrived at a tentative description and list of parameters which follow.

The device would consist of an accelerator having an orbital radius of 15 feet and an overall diameter, including the straight sections, of approximately 45 feet; the guide and focussing fields would be electrostatic, with electrode shapes as indicated in the sketch (full scale).

7

Figure 3:



Electrons of about 1 MeV energy would be injected from a small horizontal Van de Graaff generator (of the 2 MeV type manufactured by the High Voltage Engineering Corporation) so that 5% to 6% frequency modulation would be required.

Use of a reasonably large radius helps the radio frequency and observing equipment in frequency range where good techniques exist, and permits high n-values which are necessary for strong alternating-gradient focussing. (This, and phase transition, will not be modeled in the Cornell machine.) A moderate rise rate, consistent with attainable vacuum requirements, still permits the use of small, air-cooled amplifier tubes and a heavily loaded low-Q rf cavity.

A tentative list of parameters is:

Radius of curvature	15 ft
Over-all diameter	45 ft
n	200
No. of periods	37
No. of straight sections	74
No. of lenses per period	4
Length of lens	7.6 in.
Length of straight section	7.6 in.

Figure 4:

Field strength (magnetic type)	
at injection	10.5 gauss
at 10 MeV	74 gauss
Field strength (electrostatic type)	
at injection	3 kV/cm
at 10 MeV	22 kV/cm
Rise time	.01 sec
Phase transition energy	2.8 MeV
Frequency (final)	7 mc
Frequency change	54 %
Volts/turn	150 V
RF power	about 1 kw
No. of betatron wavelengths	about 6.2
aperture	1 X 1 in.
Betatron amplitude for 10^{-3} rad. error	0.07 in.
Maximum stable amplitude, synchrotron osc.	-0.16 in.
Radial spacing of betatron resonances	about 0.4 in.
Vacuum requirement	about 10^{-6} mm Hg

Total power requirements will be small and available with existing installations. The test shack seems to be a suitable location since the ring will be erected inside a thin magnetic shield which can be thermally insulated and heated economically.

We estimate the cost to be approximately $600,000, distributed as shown in the following table:

Model	Direct	Overhead	Total
Staff S. & W.	$135,000	$ 65,000	$200,000
Van de Graaff	70,000	-	70,000
Other E. & S.	130,000	-	130,000
Shops	135,000	65,000	200,000
	$470,000	$130,000	$600,000

Figure 5:

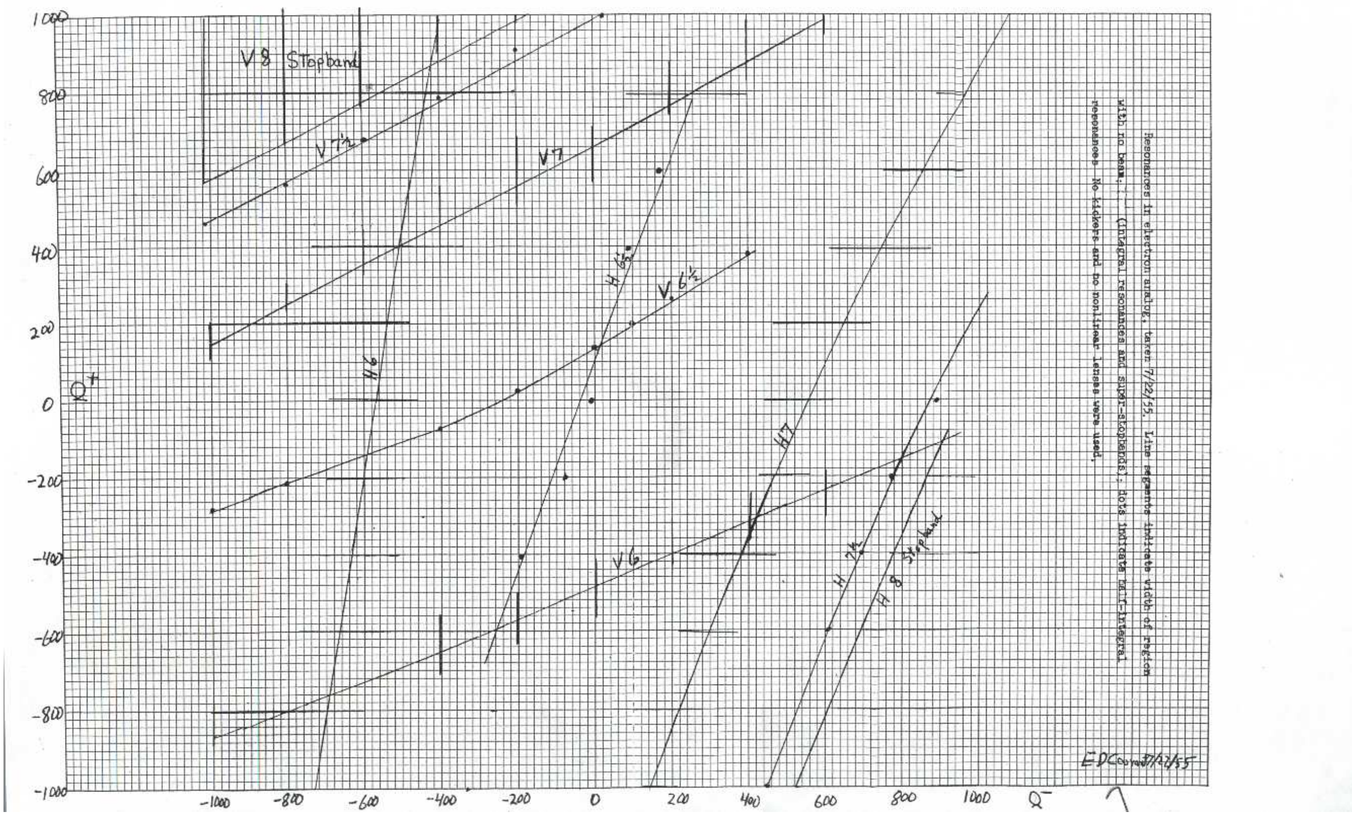


Figure 6: Tune scans actually performed on the AGS-Analogue ring and reported by Ernest Courant in a July 28, 1955 BNL technical report EDG-20. The axes are voltages (proportional to focusing strengths) applied to the tune-adjusting quadrupole families. Short heavy lines indicate regions with no beam survival (presumably due to integer resonance). Dots indicate points showing "the characteristic double envelope of the oscilloscope pattern, sometimes accompanied by beam loss" (presumably due to "very narrow" 1/2 integer resonance). The nominal central tunes values are $(Q_x, Q_y) = (6.5, 6.5)$. Stop bands due to the eightfold lattice symmetry are also shown. This figure is to be compared with Figure 8.

A.3 Reconstructed AGS-Analogue Lattice

My AGS Analogue lattice reconstruction is shown in Figure 7. As mentioned previously the reconstruction is based almost entirely on the Haworth letter, along with hints from the Plotkin report[16]. The ring is shown as having two RF cavities, diametrically-opposite, north and south. After the analysis in the rest of this paper had been completed I realized the actual ring had a single RF system in the south location, and the lattice was significantly altered nearby. The analysis in the present paper is unfaithful in this regard, but I expect little effect from this.

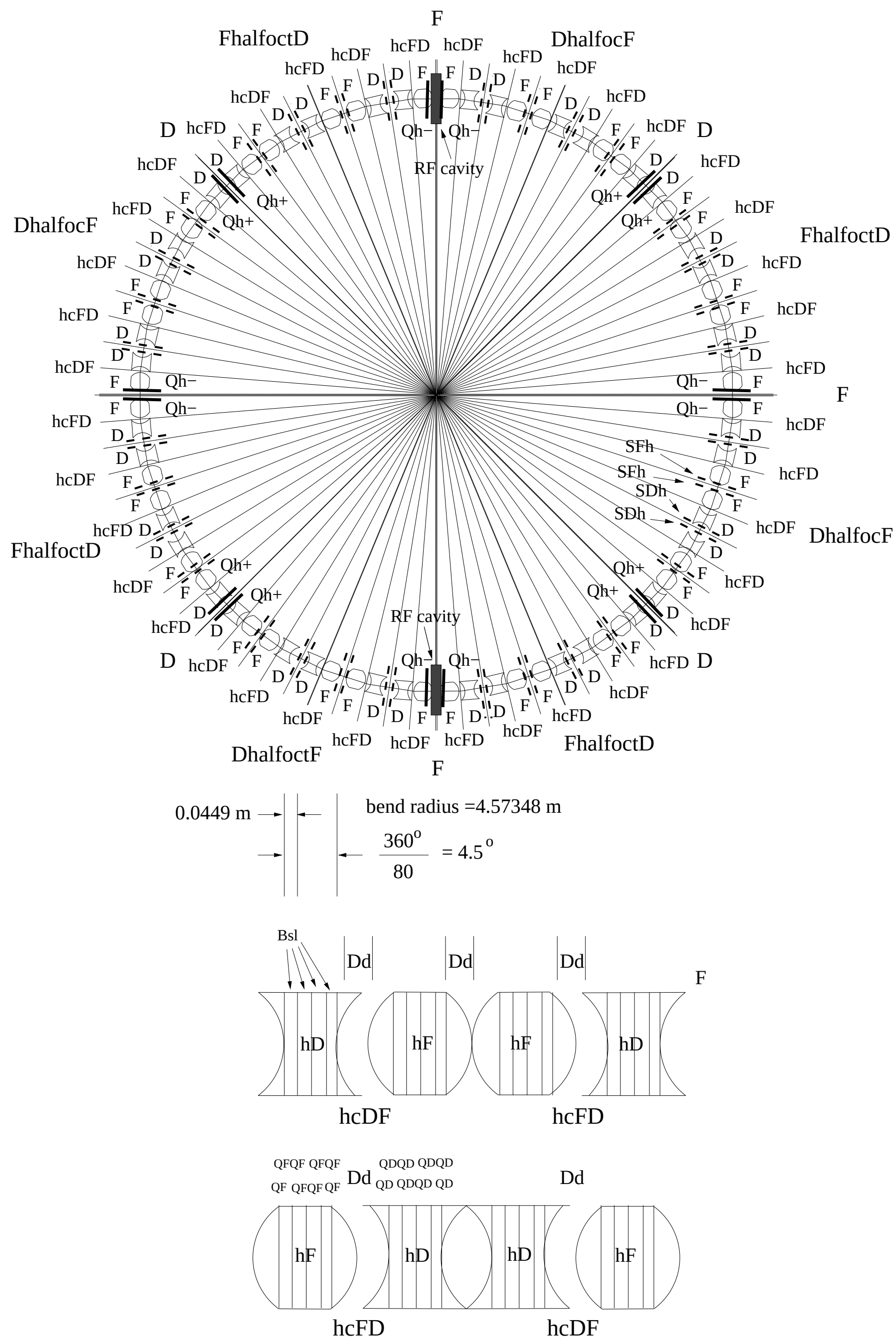


Figure 7: The 1955 AGS-Analogue lattice as reverse engineered from available documentation—mainly the 1953 proposal letter from BNL Director Haworth to AEC Director of Research Johnson.

A.4 Current Day Simulation of 1955 Machine Studies Tune Plane Scan

The tune plane resonance structure of the AGS Analogue lattice, as simulated by TEAPOT, is shown in Figure 8. This graph is to be compared with Courant's data plotted in Figure 6. There are no significantly free parameters in this comparison, though the central tunes of the simulation have been adjusted by less than ten percent to match the central tunes. After this adjustment the agreement is excellent. As it happens this is not a very sensitive test of electron codes since, for such high tunes, the difference between electic and magnetic bending is quite minor. (Otherwise it would not have been legitimate to refer to the ring as an AGS Analogue.)

For the eventual proton EDM ring the vertical tune has to be reduced from $Q_y = 6.5$ by at least a factor of ten. Comparison between all-magnetic and, otherwise identical, all-electric lattices are contained in Figures 9 and 10, for tune values $(Q_x, Q_y) = (6.2, 2.25)$. The electric/magnetic difference is still quite small, but big enough that my rudimentary tune-up tools have not yet let me reduce Q_y further.

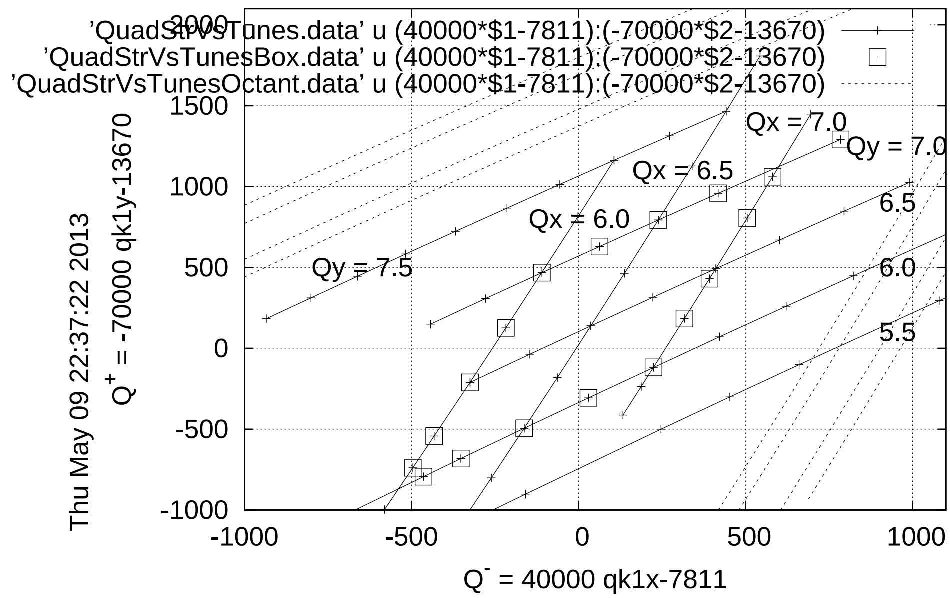


Figure 8: Tune scans using TEAPOT. Boxes indicate points on integer resonance boundary curves of the stable diamond centered on nominal tune values $(Q_x, Q_y) = (6.5, 6.5)$. Points lying on 1/2 integer resonance lines are indicated by dots. Superperiodicity (eightfold periodicity) causes resonances with $Q_x = 8$ or $Q_y = 8$ indicated by broad blank bends bounded by narrowly-spaced lines. Courant refers to these as "stop bands". $qk1x/qk2x$ are strengths of "virtual" quadrupoles representing the focusing caused by saddle-shaped/toroidal electrodes. Horizontal/vertical axes (Q^+, Q^-) are "electrode voltages on the quadrupoles in odd/even-numbered tanks". This figure is to be compared with Figure 6. The relations giving (Q^+, Q^-) as functions of $(qk1x, qk2x)$ were determined empirically, meaning that absolute focusing strength scales are not checked. Otherwise there have been no significantly adjustable lattice parameters.

A.5 Effect on Lattice Functions of Change from Magnetic to Electric Elements

Lattice functions for lattices identical except for electrical rather than magnetic elements are shown in Figures 9 and 10. Such differences only become important for low tune values. Here, with Q_y reduced to 2.25, the differences are still not very significant. They are great enough, though, that switching from magnetic to electric without compensating changed focusing typically causes a stable lattice to become unstable.

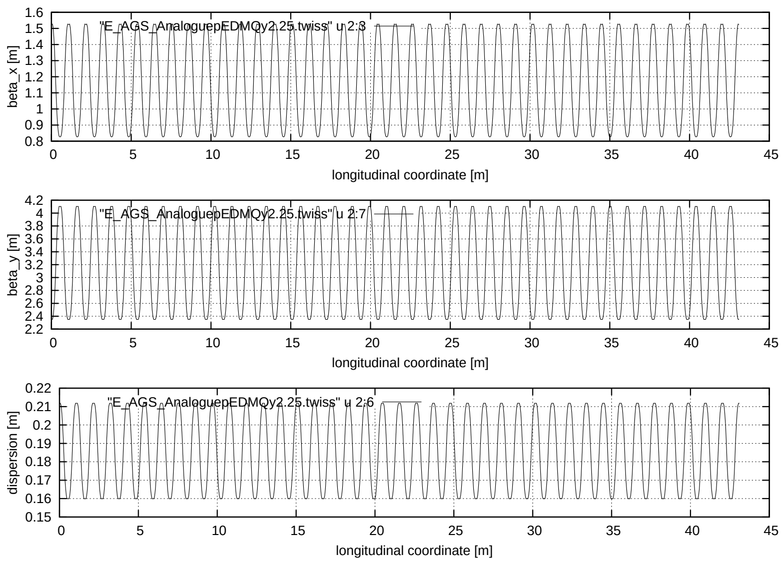


Figure 9: Lattice functions with all elements in `AGS_AnaloguepEDMQy2.sxf` treated as **magnetic**. The tunes are $Q_x = 6.2$, $Q_y = 2.25$ which is a major stepping stone in evolving from the nominal AGS Analogue tunes of $Q_x = 6.5$, $Q_y = 6.5$ to the needed EDM tunes of $Q_x = 6.2$, $Q_y = 0.2$.

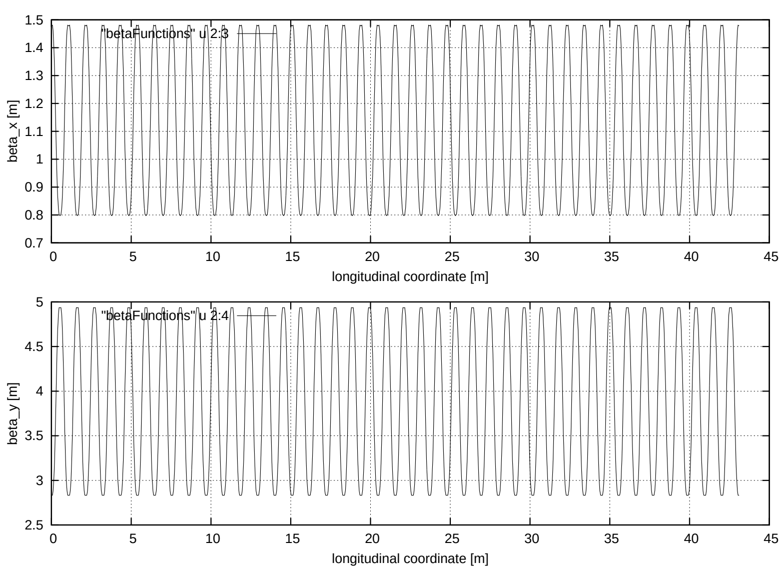


Figure 10: Lattice functions with all elements in `AGS_AnaloguepEDMQy2.sxf` treated as **electric**. All other parameters are the same as for Figure 9. Electrical effects make tuning to low Q_y difficult (without the tuning tools available for magnetic lattices).

B Later Generation, Siberian Snake Possibilities

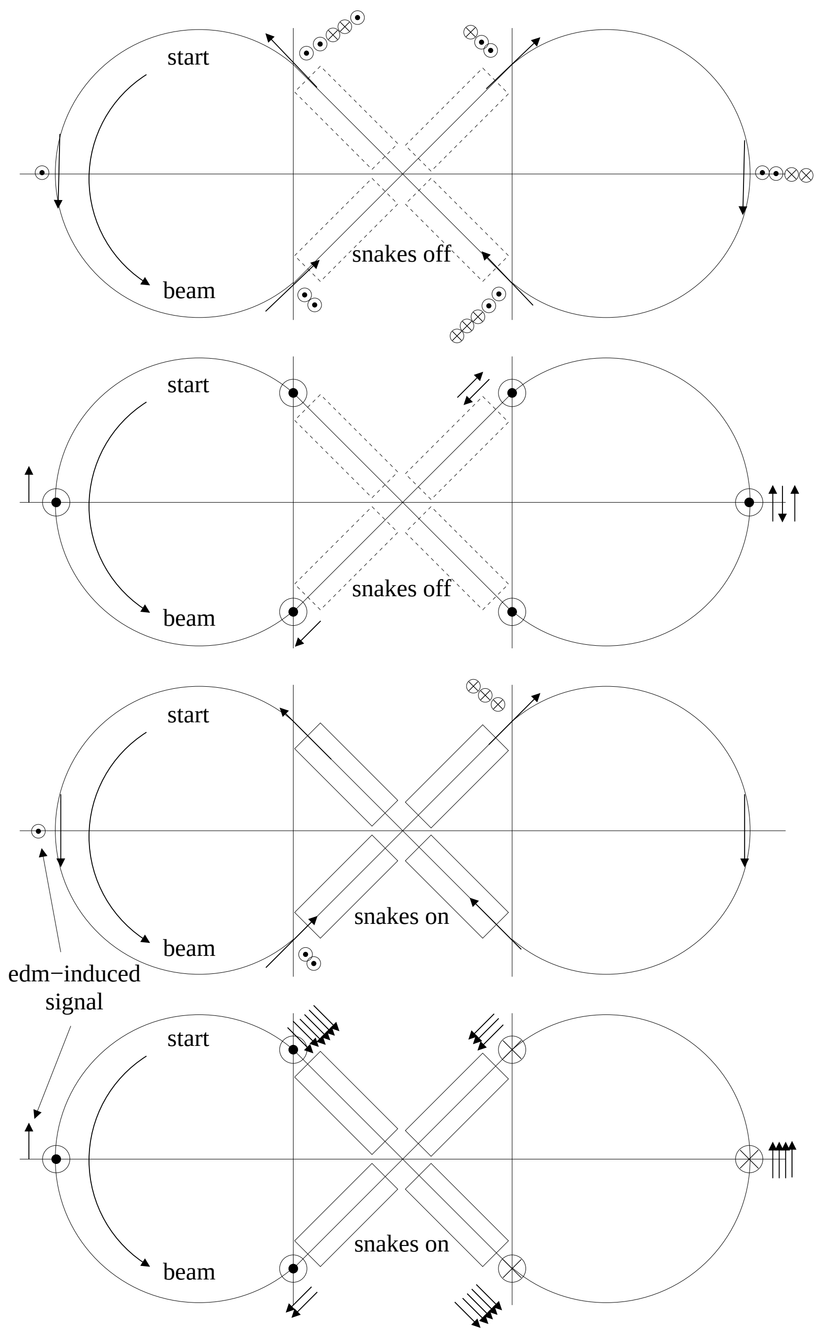


Figure 11: With snakes on the EDM signal accumulates. With snakes off the EDM signal does not accumulate.

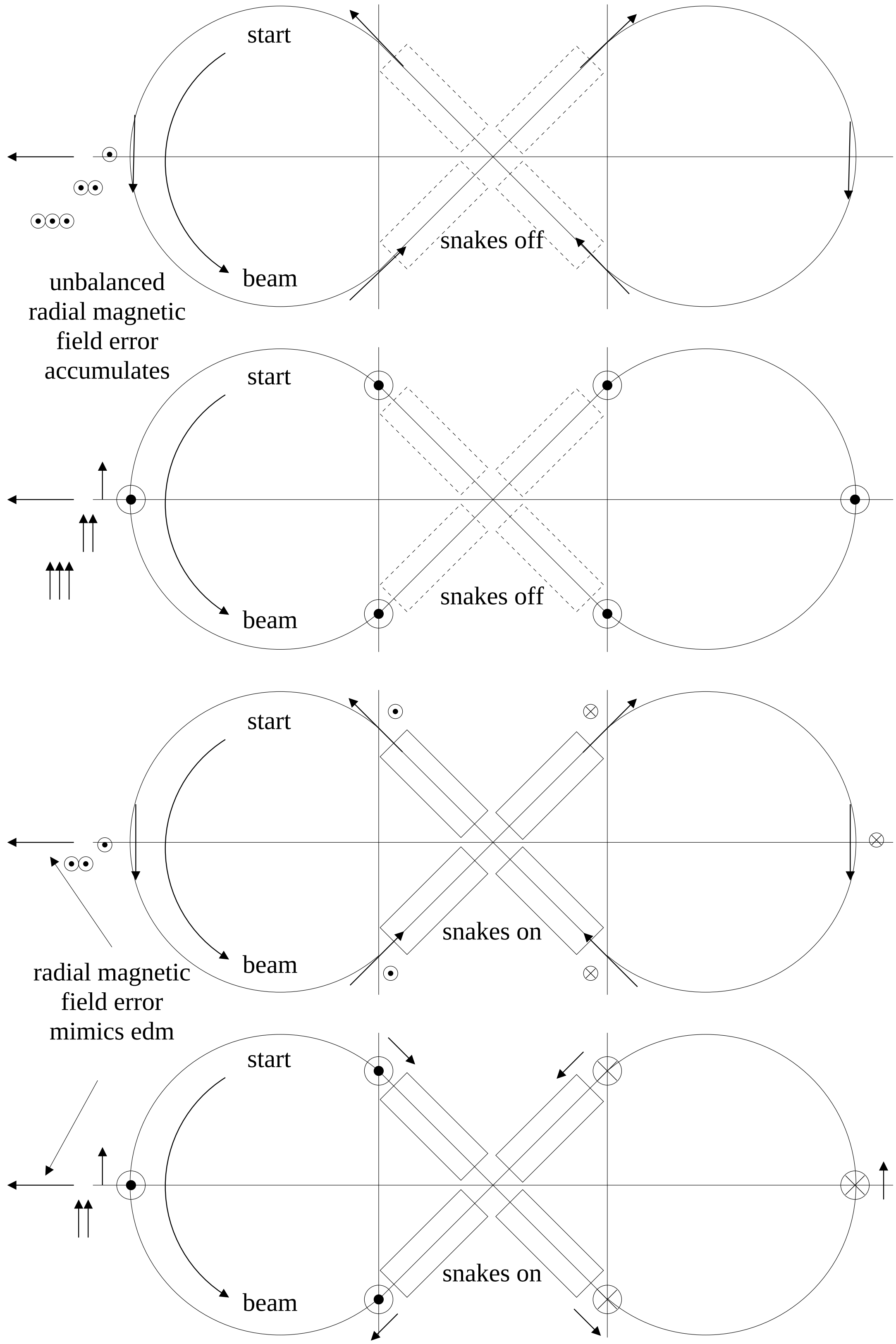


Figure 12: With snakes on the spurious "balanced" precession accumulates. With snakes off, only the unbalanced spurious precession accumulates.

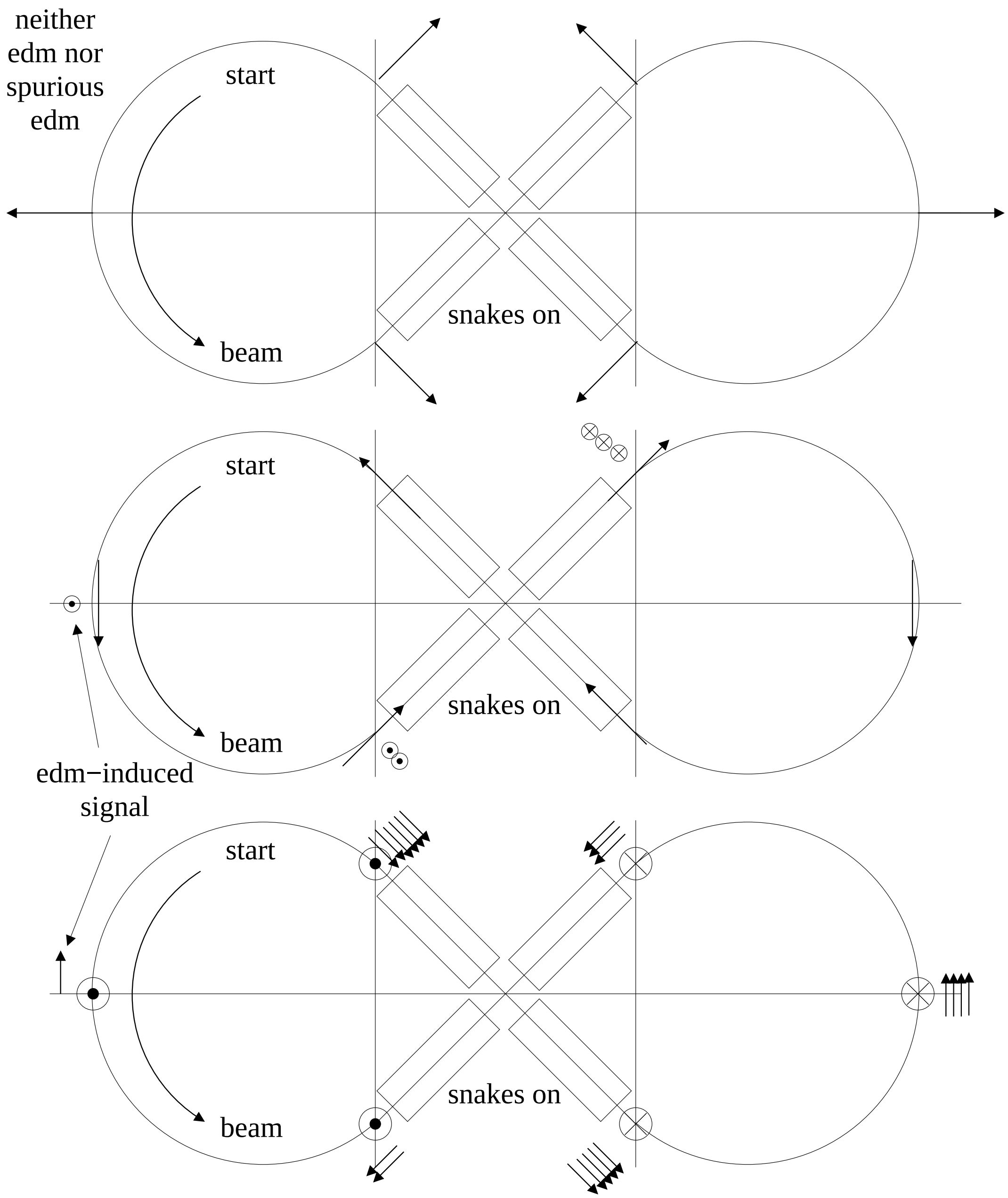


Figure 13: Only S_y an S_z polarization components are affected by the design guide field.

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